Strategic and Intensive Interventions in Mathematics - Part 1 of 2

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The Pennsylvania Training and Technical Assistance Network

#psychedaboutmath

Highlights:

Part 1

Diagnostic Assessment for Intervention Selection

Instructional Hierarchy

Intensification and features of math interventions

Acquisition Interventions

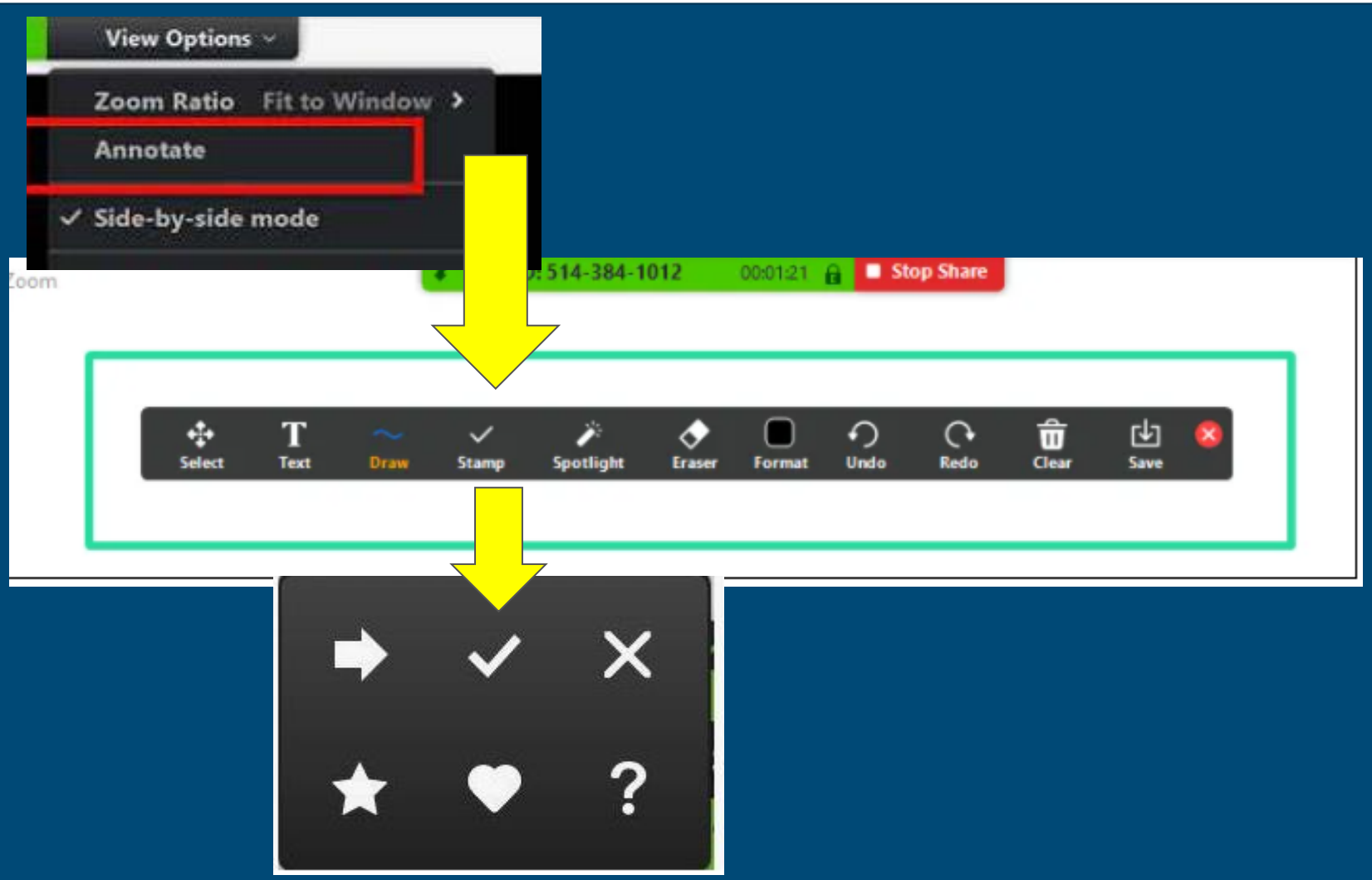
Part 2

Fluency Interventions

Progress Monitoring

Decision Making

Let’s Stamp!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Let’s practice!

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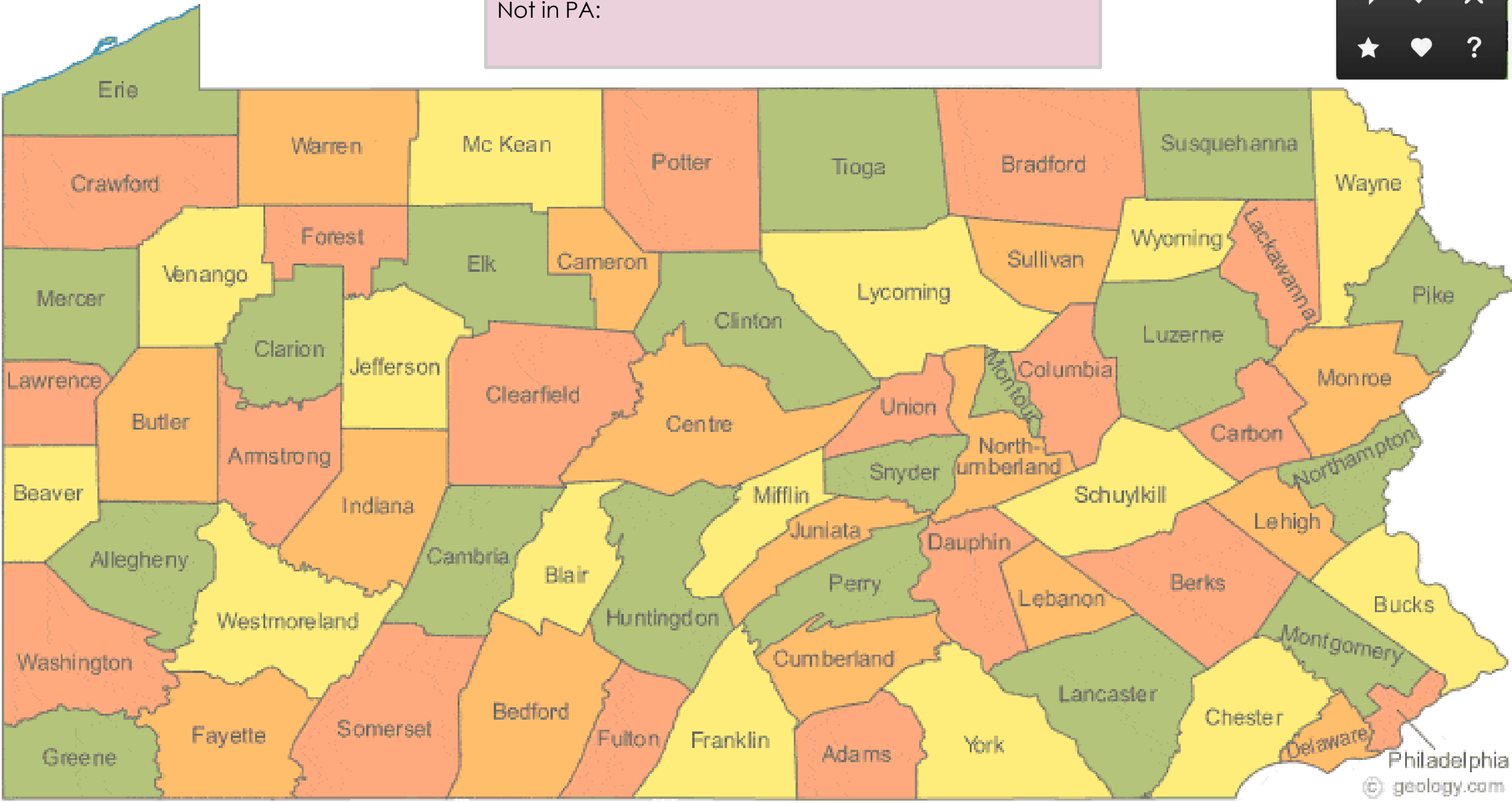
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Westmoreland
Huntingdon
Montgomery
Washington
Cumberland
Lancaster
Chester
Bedford
Somerset
Fayette
Fulton
Franklin
Adams
York
Delaware
Greene
Philadelphia
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What best describes your role? You may stamp more than one if it applies to you.

Graduate Student	Higher Ed Faculty	K-12 School Psych Practitioner	Specialized Setting School Psych Practitioner	Administrator	Other

Diagnostic Assessment

& Skill by Treatment interaction

TYPES OF DIAGNOSTIC MEASURES FOR MATH

GENERAL OUTCOME MEASURE (GOM)*

Curriculum-Based Measurement- Computation or Math Concepts and Applications *there really is not a great GOM in math like in reading*

Skills-Based Measures

Math Computation	Math Concepts and Applications
Addition Subtraction Multiplication Division	Early Numeracy (e.g., number sense) Operations and algebraic thinking Numbers and operations in base ten Measurement and data Geometry Numbers and operations-fractions Expressions and equations

Subskill Mastery Measures

Quantity Comparison 0-9 Quantity Comparison 9-19 Quantity Comparison 0-20 Quantity Comparison 20-99 Quantity Comparison 101-999 Quantity Comparison 1001-9999	Sums to 6 Sums to 12 Sums to 20 Subtraction 0-5 Subtraction 0-9 Subtraction 0-12 Subtraction 0-15 Subtraction 0-20	Fact Families Addition & Subtraction 0-5 Fact Families Addition & Subtraction 0-9 Fact Families Addition & Subtraction 0-20	2-digit Addition without regrouping 2-digit Addition with regrouping 3-digit Addition without Regrouping 3-digit Addition with and without Regrouping 2-digit Subtraction without Regrouping	2-digit Subtraction with Regrouping 3-digit Subtraction without Regrouping 3-digit Subtraction with Regrouping 3-digit Subtraction with and without Regrouping 3-digit Addition & Subtraction with and without Regrouping

Sequence Skills

What are the prerequisite skills?

Retention of previously learned skills improves when a small amount of material is presented at a time.

- **What are the foundational skills?**
- **Too many skills or difficult tasks increase off-task behavior.**

Task Analyses

How can the concepts be divided into smaller units?

Students learn each skill to mastery from the earliest unknown skill.

- **Simpler skills + concepts come first**

Subskill Mastery

As students master subskills, level of teacher help fades.

Match student performance to instructional hierarchy.

- **Acquisition**
- **Fluency**
- **Maintenance/ Generalization**

Example of Sequence of Skills: Acadience Math (1st grade)

Might we need to break down any of these skills into smaller units?

Acadience Math–First Grade Computation

Problem #	Problem Description
1, 11	Add 0 or 1 to a one-digit number.
2, 15, 20, 22	Add two one-digit numbers, excluding 0 and 1.
3, 13	Subtract a one-digit number from a two-digit number (within 20) resulting in a difference of 11 or more, without renaming.
4, 10, 21, 24	Subtract a one-digit number from a one-digit number, excluding 0 and 1 in the subtrahend.
5, 17	Subtract a one-digit number from a two-digit number (within 20), resulting in a difference of 9 or less.
6, 18	Subtract 0 or 1 from a one-digit number.
7, 12	Add a two-digit and a one-digit number, with renaming, resulting in a sum of 20.
8, 14	Subtract a one-digit number from 20, with renaming.
9, 16, 19, 23	Add a two-digit and a one-digit number, without renaming, resulting in a sum of 20 or less.

Where might I find a sequence of skills? Check out: *CBA Manual*

(Wright, Appendix D)

M I D M=mastered I=Instructioned D=Difficulty

Grade 1

__ __ __ 1. Add two one-digit numbers: sums to 10.

__ __ __ 2. Subtract two one-digit numbers: combinations to 10.

Grade 2

__ __ __ 3. Add two one-digit numbers: sums 11 to 19.

__ __ __ 4. Add a one-digit number to a two-digit number--no regrouping.

__ __ __ 5. Add a two-digit number to a two-digit number--no regrouping.

__ __ __ 6. Add a three-digit number to a three-digit number--no regrouping.

__ __ __ 7. Subtract a one-digit number from a one- or two-digit number: combinations to 18.

__ __ __ 8. Subtract a one-digit number from a two-digit number--no regrouping.

__ __ __ 9. Subtract a two-digit number from a two-digit number--no regrouping.

__ __ __ 10. Subtract a three-digit number from a three-digit number--no regrouping.

__ __ __ 11. Multiplication facts--0's, 1's, 2's. column.

__ __ __ 20. Add a three-digit number to a three-digit number with regrouping from the units to the tens column only.

__ __ __ 21. Add a three-digit number to a three-digit number with regrouping from the tens to the hundreds column onl

Adapted from Shapiro (1989)

Sequence of Skills?

Sums to 6	Sums to 12	Sums to 20	Subtraction 0-5	Subtraction 0-9	Subtraction 0-12	Subtraction 0-15

Find out where **student is** … find out where student **needs to be.**

Calculating DCPM

- Digits correct per minute can be used as a metric to score student mathematics performance (computation).

- Helps provide a consistent metric (vs. problems correct) to show incremental growth (e.g., student doesn’t understand concept of regrouping yet and gets 0 digits correct, but then learns how to regroup ones and gets partial digits correct in a 2-digit + 2-digit addition problem).

  Error analysis when scoring DCPM can provide insight on the types of errors students are making on math probes, and better assist us in making an effective instructional match.

Putting DCPM in Context:

Norms, skill, administration time, growth

CBA for Computation (Adapted from Codding, Volpe & Poncy, 2017)

Performance Level	Deno & Mirkin (1977)	Howell & Nolet (1999)	Burns, VanDerHeyden & Jiban (2006)	Stage of Instructional Hierarchy
Frustrational	Grades 1-3 <10 DCPM Grades 4+ < 20 DCPM	<20 DCPM	Grades 2-3 <14 DCPM Grades 4-5 <24 DCPM	Acquisition
Instructional	Grades 1-3 10-19 DCPM Grades 4+ 20-39 DCPM	20-30 DCPM	Grades 2-3 14-31 DCPM Grades 4-5 24-49 DCPM	Fluency
Mastery	Grades 1-3 20+ DCPM Grades 4+ 40+DCPM	40+ DCPM	Grades 2-3 32+ DCPM Grades 4-5 50+ DCPM	Maintenance; Generalization; Adaptation

TABLE 7.2. Subskills for Number Foundations

Early numeracy assessments by skill area	Assessment	Skill	Time	Mastery
Gateway skills: A foundational piece to understanding quantity is acquiring gateway skills. These include counting skill, number recognition, and writing numbers. These skills are needed to support the development of concepts aligned with number sense such as correspondence, cardinality, and quantitative relationships.	Oral counting	Rote counting	1 minute	80 DCPM
	Number identification	Symbol recognition	1 minute	60 DCPM
	Number writing	Symbol production	1 minute	60 DCPM
Number Sense: Students develop skills to describe, compare, and manipulate numbers and their associated quantities. Students expand mathe0related vocabulary to perform and describe these tasks across contexts.	Missing number	Counting from number	1 minute	20 DCPM
	Quantity discrimination	Number comparison	1 minute	20 DCPM
Number combinations: Students combine gateway and number sense skills, and write and solve basic number sentences to represent the manipulation of quantity.	Number combinations: addition	Addition: Sums to 10	1 minute	20 DCPM
	Number combinations: Subtraction	Subtraction: Sums from 10	1 minute	20 DCPM

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TABLE 7.3. Subskills for Basic Facts and Procedural Computation

Probe set	Skill	Problem type(s)	Time	Mastery
Basic fact computation: These single-skill probes consist of basic number combinations.	+	0 + 0 to 9 + 9	1 minute	40 DCPM
	-	0 - 0 to 18 - 9 single-digit answers	1 minute	40 DCPM
	x	0 x 0 to 9 x 9	1 minute	40 DCPM
	÷	0 ÷ 0 to 81 ÷ 9 single-digit answers	1 minute	40 DCPM
Mixed computation: These probes target a single skill (e.g., addition) and combine operations with the targeted skill.	+	Mixed addition	3 minute	20 DCPM
	-	Mixed subtraction	3 minute	20 DCPM
	x	Mixed multiplication	3 minute	20 DCPM
	÷	Mixed division	3 minute	20 DCPM

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TABLE 7.4. Subskills for Procedural Computation

Skill		Problem type(s)	Time	Mastery
Multidigit addition	+	3 x 3 digit	2 minutes	20 DCPM
	+	2 x 1 digit with regrouping	2 minutes	20 DCPM
	+	3 x 3 digit with regrouping	2 minutes	20 DCPM
Multidigit subtraction	-	3 x 3 digit	2 minutes	20 DCPM
	-	2 x 1 digit with regrouping	2 minutes	20 DCPM
	-	3 x 3 digit with regrouping	2 minutes	20 DCPM
Multidigit multiplication	x	2 x 1 digit	2 minutes	20 DCPM
	x	4 x 2 digit	2 minutes	20 DCPM
	x	4 x 3 digit	2 minutes	20 DCPM
Multidigit division	÷	3 ÷ 1 digit no remainder	2 minutes	20 DCPM
	÷	3 ÷ 1 digit with remainder	2 minutes	20 DCPM
	÷	3 ÷ 1 digit with decimals	2 minutes	20 DCPM

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Growth: From Wright (2013) *Intervention Central*

www.interventioncentral.org

Grade	End of Year Benchmark: Correct Digits per 2 Mins (Fuchs & Fuchs, n.d.)	Weekly Growth: 'Realistic' (Fuchs & Fuchs, 1993)	Weekly Growth: 'Ambitious' (Fuchs & Fuchs, 1993)
1	20	0.3	0.5

Grade	Performance Level	Correct Digits per 1 Min (Burns, VanDerHeyden, & Jiban, 2006)	Weekly Growth: 'Realistic' (Fuchs & Fuchs, 1993)	Weekly Growth: 'Ambitious' (Fuchs & Fuchs, 1993)
2	Mastery	More than 31	0.3	0.5
	Instructional	14-31		
	Frustration	Less than 14		
3	Mastery	More than 31	0.3	0.5
	Instructional	14-31		
	Frustration	Less than 14		
4	Mastery	More than 49	0.75	1.2
	Instructional	24-49		
	Frustration	Less than 24		
5	Mastery	More than 49	0.75	1.2
	Instructional	24-49		
	Frustration	Less than 24		

Grade	Performance Level	Correct Digits per 2 Mins (Deno & Mirkin, 1977)	Weekly Growth: 'Realistic' (Fuchs & Fuchs, 1993)	Weekly Growth: 'Ambitious' (Fuchs & Fuchs, 1993)
6	Mastery	More than 79	0.45	1.0
	Instructional	40-79		
	Frustration	Less than 40		

Recent Research (2020) Growth Rates on ROI DCPM: Single-Case Designs

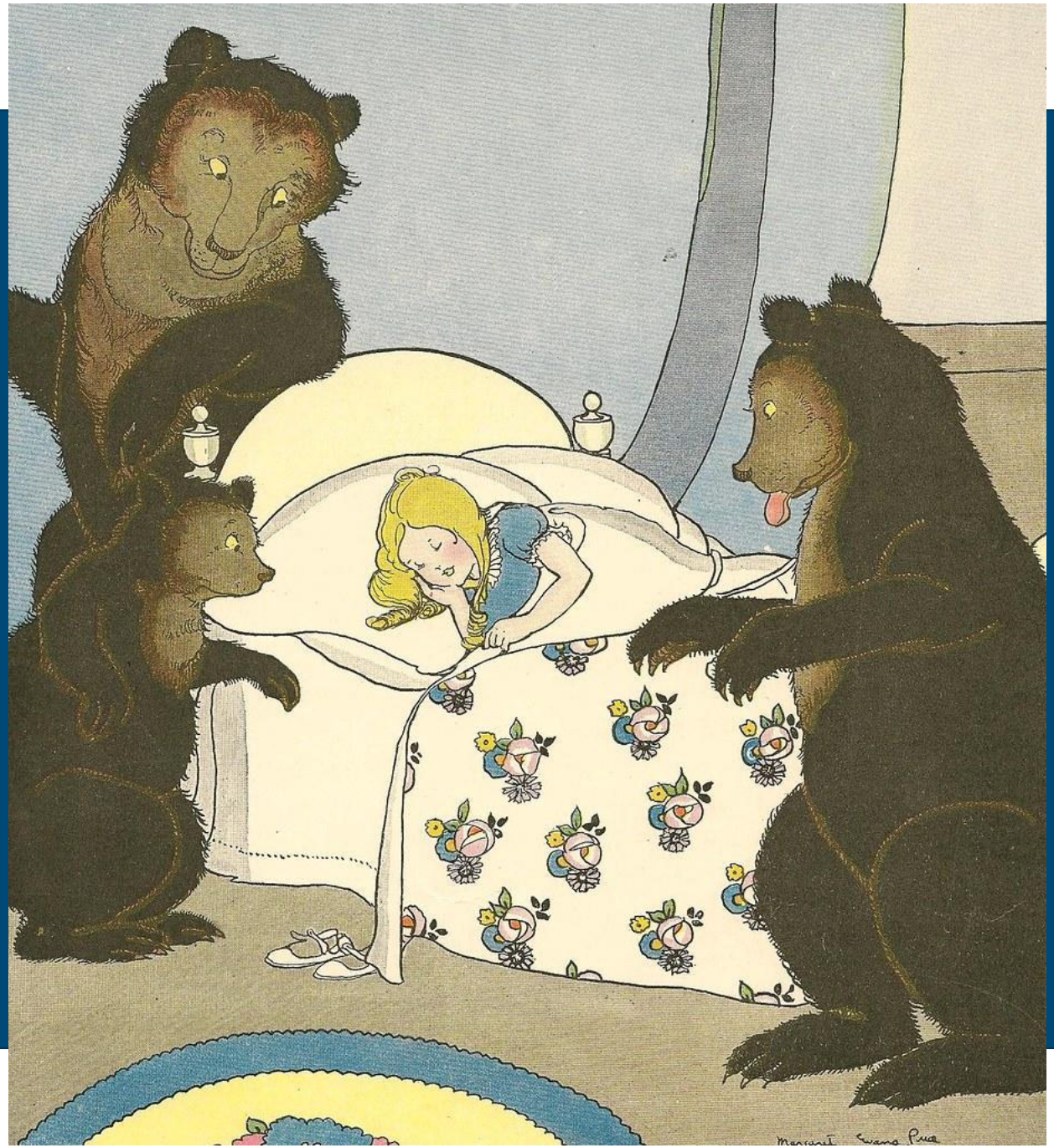
It was found that the average growth rate across sessions of intervention was 1.62 DCPM, and across blocks of 15 min of instructional time was 3.58 DCPM for single-case research designs.

Interventions Included: Cover Copy Compare, Concrete, Representational Abstract, Detect Practice Repair, Explicit Timing, Math to Mastery, Taped Problems

Solomon, B.G, Poncy, B.C., Batista, C., & Campana, K.V. (2020). A review of common rates of improvement when implementing whole number operation math interventions, *School Psychology*, 35(5), 353-362.

Instructional Hierarchy

A Foundation for Intervention Matching



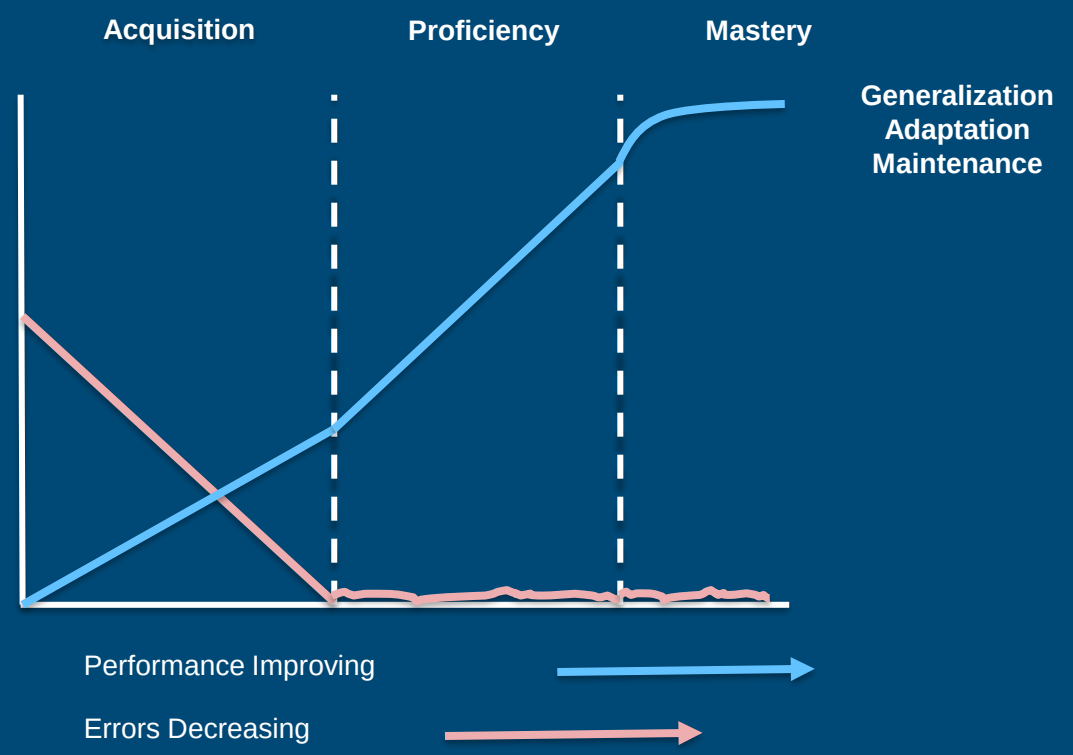
<table>
<tr><th></th><th>Acquisition</th><th>Proficiency</th><th>Generalization</th><th>Adaption</th></tr>
<tr><td>IF

<u>Learning Hierarchy</u></td><td rowspan="2">Haven’t had enough help</td><td rowspan="2">Haven’t had enough practice</td><td colspan="2" rowspan="2">Haven’t had to do it that way before</td></tr>
<tr><td>THEN

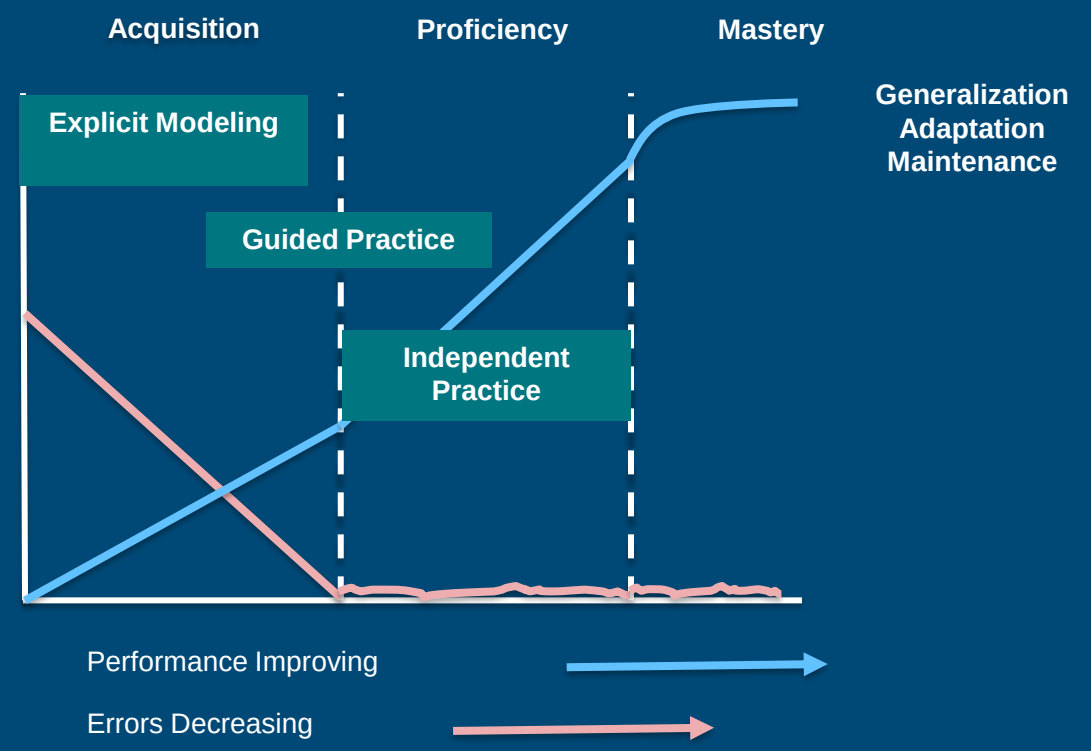
<u>Instructional Hierarchy</u></td></tr>
</table>

Adapted from…Haring, N. G., & Eaton, M. D. (1978). Systematic instructional procedures: An instructional hierarchy. In N. G. Haring, T. C. Lovitt, M. D. Eaton, & C. L. Hansen (Eds.) *The fourth R: Research in the classroom* (pp. 23-40). Columbus, OH: Charles E. Merrill.

Stages of Learning Computation



Stages of Learning Computation



What strategies are effective for students in the acquisition stage?

- Teacher actively demonstrates target skill
- Teacher uses 'think-aloud' strategy-- especially for thinking skills that are otherwise covert
- Student has models of correct performance to consult as needed (e.g., correctly completed math problems on board)
- Student gets feedback about correct performance
- Student receives praise, encouragement for effort

("The Instructional Hierarchy: Linking Stages of Learning to Effective Instructional Techniques | Intervention Central," 2020)

What strategies are effective for students in the fluency stage?

- Teacher structures learning activities to give student opportunity for active (observable) responding
- Student has frequent opportunities to drill (direct repetition of target skill) and practice (blending target skill with other skills to solve problems)
- Student gets feedback on fluency and accuracy of performance
- Student receives praise, encouragement for increased fluency

("The Instructional Hierarchy: Linking Stages of Learning to Effective Instructional Techniques | Intervention Central," 2020)

What strategies are effective for students in the generalization / maintenance stage?

- Teacher structures academic tasks to require that the student use the target skill regularly in assignments.
- Student receives encouragement, praise, reinforcers for using skill in new settings, situations
- If student confuses target skill with similar skill(s), the student is given practice items that force him/her to correctly discriminate between similar skills
- Teacher works with parents to identify tasks that the student can do outside of school to practice target skill
- Student gets periodic opportunities to review, practice target skill to ensure maintenance

The Instructional Hierarchy: Linking Stages of Learning to Effective Instructional Techniques, Intervention Central(2020)

Instructional Intensification

Dimension of Intensity	Definition
Strength	How well the program works for students with intensive intervention needs (what is effect size?)
Dosage	# of opportunities a student has to respond and receive corrective feedback
Alignment	How well the program (a) addresses the target student's full set of academic skill deficits, (b) does not address skills the target student has already mastered (extraneous skills for that student), and (c) incorporates a meaningful focus on grade-appropriate curricular standards
Attention to Transfer	The extent to which an intervention is designed to help students (a) transfer the skills they learn to other formats and contexts and (b) realize connections between mastered and related skills
Comprehensiveness	# of explicit instruction principles the intervention incorporates
Behavioral Support	program incorporates (a) self-regulation and executive function components and (b) behavioral principles to minimize non-productive behavior
Individualization	A validated, data-based process for process for individualizing intervention, with which the special educator systematically adjusts an intensive intervention platform over time to address the student's complex learning needs

Adapted from: Fuchs, Fuchs & Malone (2017), *The Taxonomy of Intervention Intensity, Teaching Exceptional Children (50), 35-43.*

When do we intensify intervention?

Student need persists despite:

- Intervention implementation with fidelity and for appropriate frequency/duration
- Evidence of effectiveness with similar students
- Use of a valid and reliable progress-monitoring tool that is sensitive to student change and matched to the correct measurement outcome

Intervention Alignment to Grade-Level Curriculum Tips: National Center on Intensive Intervention

Intervention is aligned to grade-level curriculum if it:

Provides instruction for prerequisite skills required for content taught in core instruction.

Allows students to practice the grade-level skill on instructional-level text.

Provides more opportunities to respond and receive feedback on grade-level skills.

An intervention would **not** be aligned to the grade-level curriculum if it:

Provides instruction on some skills but neglects others.

Replaces core instruction so that students do not have the opportunity to develop background knowledge, vocabulary, and so on from grade-level content.

Key Features of Tier 2 & 3 Mathematics Intervention

-Explicit Instruction

-Strategy Instruction

-Sequencing Instruction

-Ongoing Progress Monitoring & Feedback

-Drill, practice & cumulative review

-Student verbalization

-Visual representation

-Motivation + Reinforcement

Codding, Volpe & Poncy (2016)

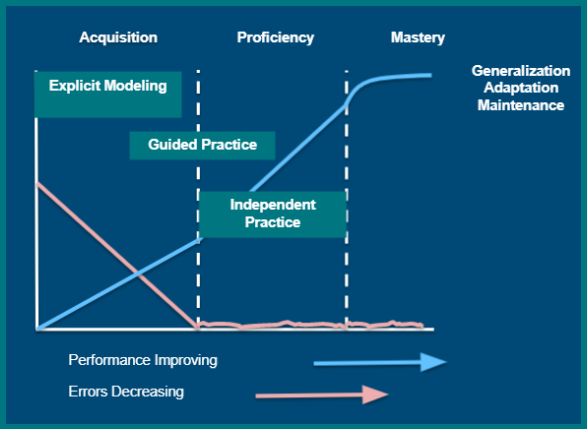
Interventions

Mathematics Interventions Aligned to Learning Stages / Instructional Hierarchy

Intervention	Skill(s) Targeted	Individual, Small Group, Classwide Intervention	Time / Dosage	Learning Stage / Instructional Hierarchy
The Great Race	Counting, cardinality (1-10)	Individual, small group	15 minutes (individual) or 25 minutes (small group) 2 x per week for 2 weeks	Fluency
Race to Space	Counting, cardinality (1-100)	Individual, small group	15 minutes (individual) or 25 minutes (small group) 2 x per week for 2 weeks	Fluency
Concrete – Representational-Abstract (CRA)	Conceptual understanding, language / vocabulary, procedural skills for early numeracy, addition, subtraction, multiplication, division, fractions, integers, equations	Individual, small group, classwide	Varied dependent upon skill	Acquisition
Incremental Rehearsal	Basic math facts (addition, subtraction, multiplication, division) Numeral identification	Individual	10-20 minutes per session 2x per week for 4-12 weeks	Acquisition - Fluency
Math to Mastery	Computation (addition, subtraction, multiplication, division)	Individual	10-minute sessions 3 to 5 times weekly for 3 to 8 weeks	Acquisition - Fluency
Drill Sandwich	Basic math facts (addition, subtraction, multiplication, division) Numeral identification	Individual *could be adapted for partner work*	10-20 minutes per session 2x per week for 4-12 weeks	Fluency
Classwide Fluency Building	Numbers and operations, computation skills, word problem solving, procedural fluency	Classwide	Approximately 10-15 minutes daily	Fluency

Cover-Copy-Compare	Computation skills	Individual, small group, classwide	3-10 minutes daily 2 to 5 sessions per week 2 to 16 weeks	Fluency
Detect-Practice-Repair	Computation skills	Individual, small group, classwide	10-12 minutes daily	Fluency *works best for students with higher fluency skills on target skill than those who are dysfluent*
Taped Problems	Math facts; simple and complex computation problems (addition, subtraction, multiplication, division)	Individual *Adapted for Classwide w/ added group contingency	8-15 minutes per session, 3 to 5 times per week for 3 to 10 weeks	Fluency
Explicit Timing	Addition, subtraction & multiplication (single and multi-digit)	Small group, classwide	3-10 minute sessions 2 to 5 times per week 4 to 6 total weeks	Fluency
High-Preference / Interspersed Problems	Complex computation including addition and subtraction with regrouping	Individual, small group, classwide	10 minutes daily 3 to 5 times per week For 3 to 5 weeks	Fluency - Generalization

Classwide intervention is most effective when delivered 4-5 times per week.



Acquisition Interventions

What best describes your interest in spending more time on each of the following? Stamp once for each:

Strategy	Full Plate!!!	Just Dessert Please	I want to go home!
CRA			
Math to Mastery			
Incremental Rehearsal			
Math Vocabulary			
Word Problems			

CRA

Concrete - Representational - Abstract

Intentionally scaffolding to make concepts/procedures more explicit

Concrete – Representational - Abstract

Acquisition

Routine

1. Develop the CRA progression
   a) Outline the thought/language requirements of the abstract problem.
   b) Select/develop modeling structure that can be used with object and easily drawn.
   c) Determine the manipulative(s) to be used.
   d) Establish connections between the three models
2. Teach Concrete Model
   a) Teach the manipulative
   b) Ensure correct language/thought process
3. Teach Representational Model
   a) Build efficiency with modeling and language
   b) Increase complexity of problems
4. Teach Abstract Model
   a) Build understanding from concrete and representational models
   b) Utilize language to think aloud
5. Ongoing Progress Monitoring
   a) Develop a task analysis in line with the math concept/procedure
   b) Assess student on the process and provide corrective feedback
   c) Transition through CRA progression as students demonstrate understanding.

Resources

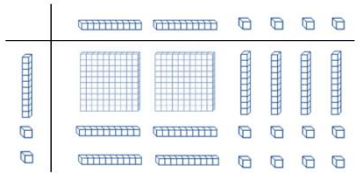
Overview
http://ebi.missouri.edu/wp-content/uploads/2013/08/EBI-Brief-Template-Concrete-Representational-Abstract.pdf

When to use?

Use this intervention if:

- Student struggles with underlying concepts
- Student struggles with math language
- Student struggles with math facts
- Student needs support with procedural skills

Example Activity



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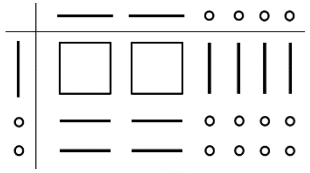

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10	200	40	240
2	40	8	+ 48
			288

A

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PaTAN

Addition Overview

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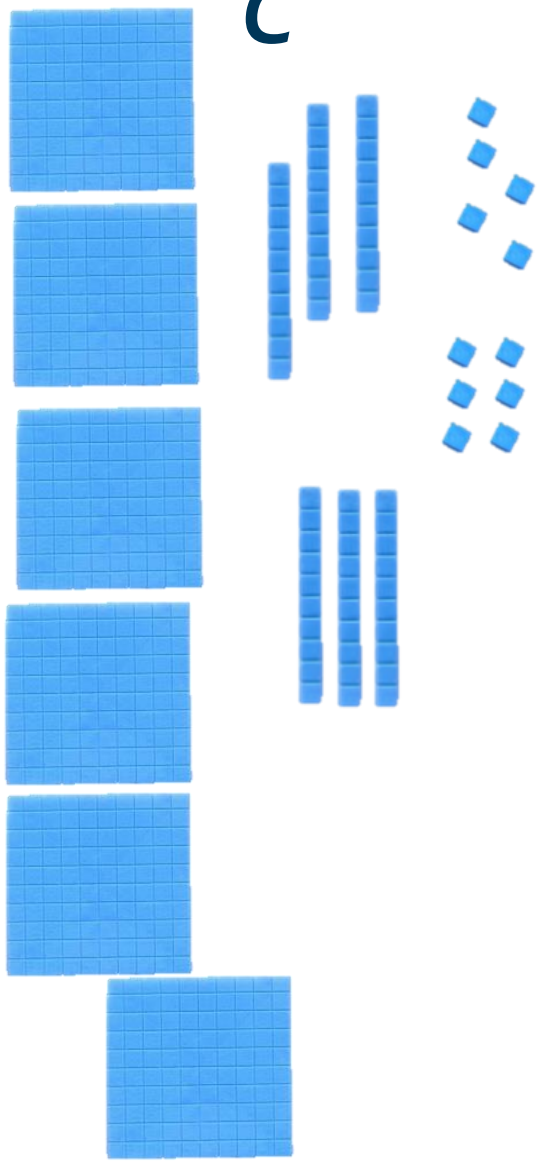

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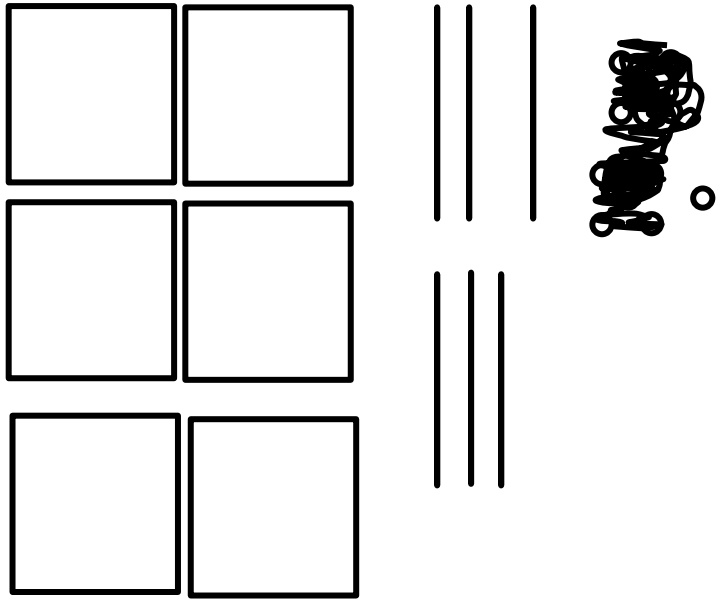

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Equivalence Overview

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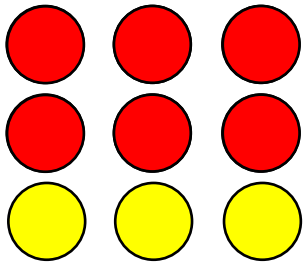

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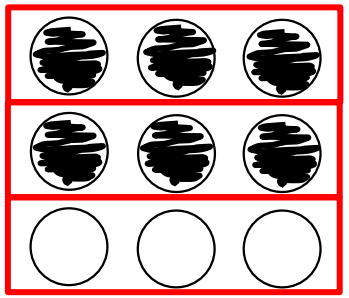

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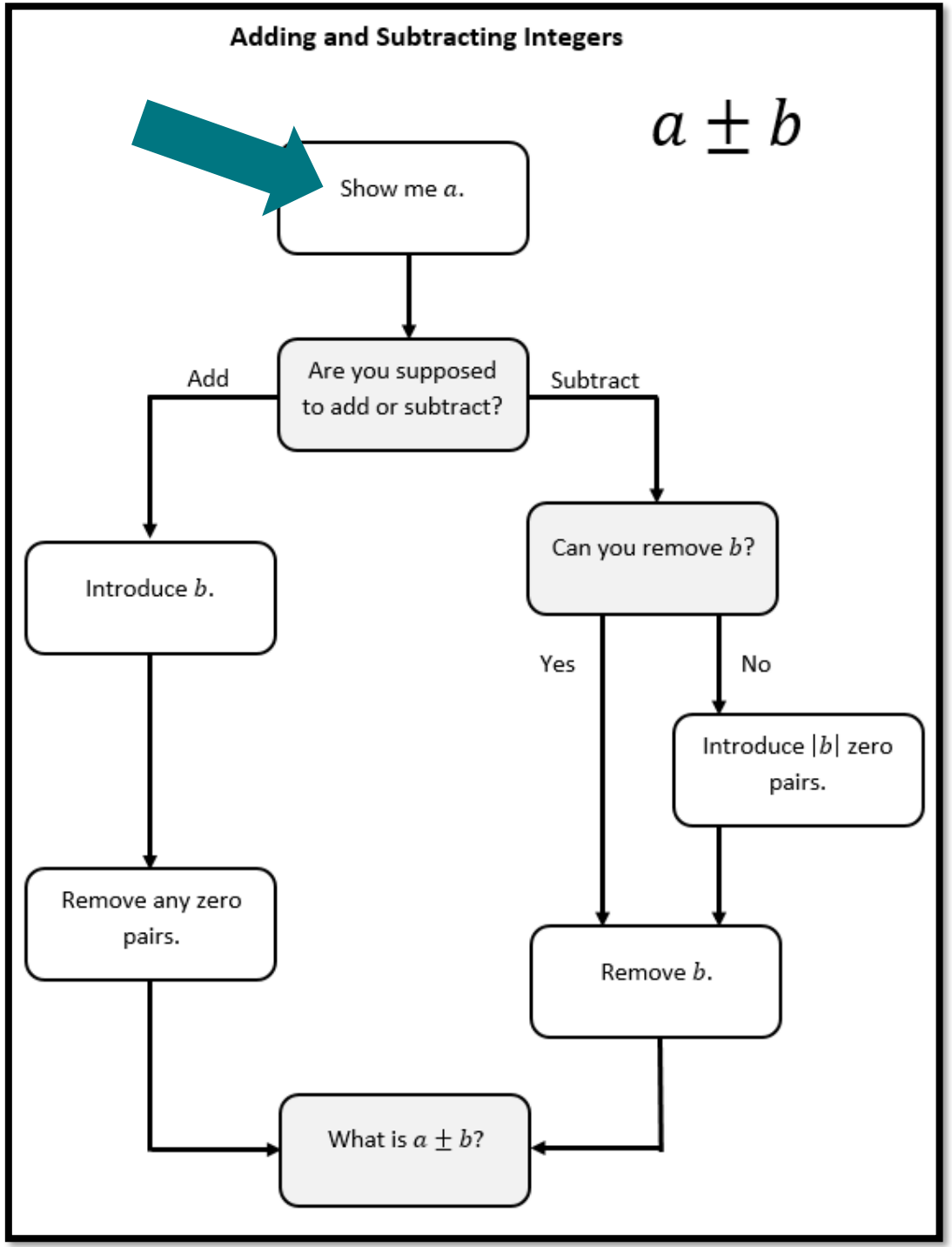
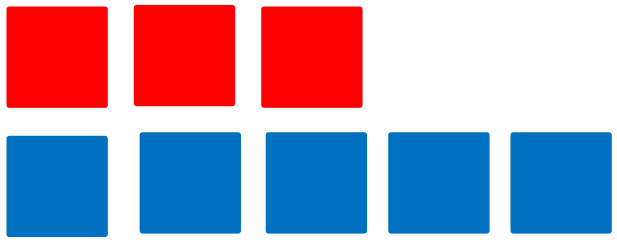
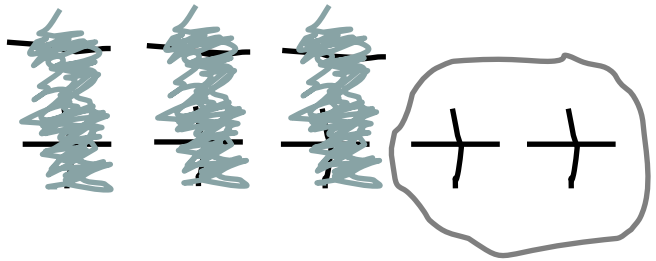


Concrete



Representation



Abstract

$$-3 + 5 = 2$$

Math to Mastery

-Teacher-directed intervention for individual students

-Computation skills (researched on addition-can be applied to other operations)

-Teacher drills down to earliest prerequisite skill for intervention

-1-minute trials to mastery (e.g., 32 dcpm) or maximum of 10 1-minute trials

Doggett, Hennington, & Johnson-Gros (2006)

Math to Mastery

Acquisition/ Fluency

Routine

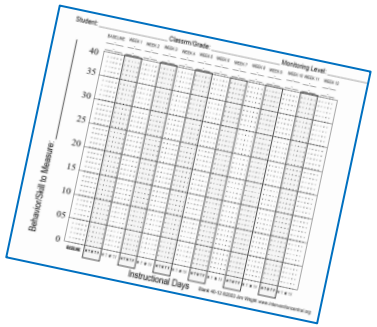
1. Determine the skill(s) used for the intervention according to CBA or computer adapted assessments.
   a) The earliest unmastered pre-requisite skills in the sequence for the math operation should be targeted first.
2. Demonstrate how to complete each math problem on a worksheet while the student follows along on his/her copy.
3. Student practices the problems on the worksheet in one-minute trials until mastery criterion (e.g., 32 DCPM) is attained or ten one-minute trials are completed.
4. Follow along while student is working, marking errors and giving immediate corrective feedback.
5. After each one-minute trial, compute the digits correct and errors; provide specific praise for effort and performance.
6. Student charts his/her performance at the end of each 1-minute trial.
7. At the end of each session (or at least once weekly), student is administered a brief assessment (e.g., CBA), which is completed individually and scored to monitor progress on the target skill.
8. Once every other week (or monthly), a multiple skill CBM probe is administered to progress monitor for generalization.

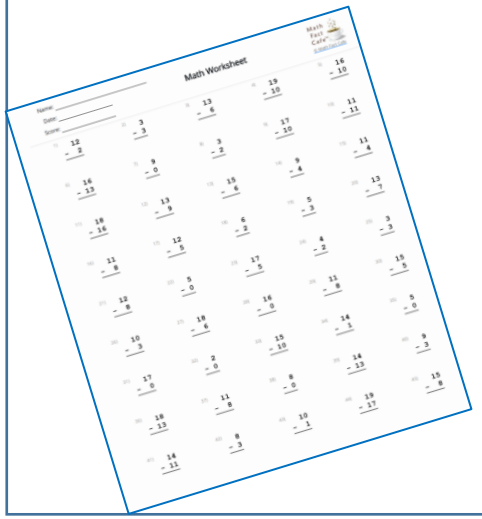
When to use?

Use this intervention if:

- Student is having difficulty with computation skills (addition, subtraction, multiplication, division)

Example Activity





Resources

https://nysrti.org/intervention-tools/math-tools/tool:math-to-mastery/

Doggett, R.A., Henington, C., & Johnson-Gros, K. N. (2006). *Math to Mastery: A direct instruction remedial math intervention designed to increase student fluency with basic math facts.* Unpublished manuscript, Mississippi State University.

PaTAN

Incremental Rehearsal

-Teacher-directed intervention for individual students

-Computation skills- acquisition moving toward fluency

Strong effect size: 1.67 (Burns, Zaslofsky, Kanive, & Parker, 2012)

-Assess for known and unknown facts

-9 knowns to 1 unknown (90% known to 10% unknown)

Incremental Rehearsal

Fluency

Routine

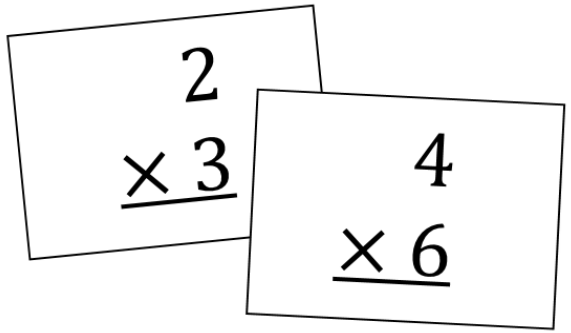
1. Develop the flashcard sets of known and unknown items
   a) Assess the student by showing them each of the cards one at a time.
   b) If the student responds orally with the correct fact within 2 seconds, place the flashcard in the known items pile
   c) If the student gives an incorrect response, no response, or a correct response after 2 seconds, place the flashcard in the unknown item pile.
   d) To prepare for the intervention, select 9 known items and 1 unknown item. The rest of the known items will not be used.
2. Run first intervention trial
   a) Present the unknown item. Read the problem and the answer. Student repeats the problem and the answer.
   b) Present a known item. Student reads the problem and answer.
      - If student errors, teacher prompts with the correct fact and student repeats. Process is continued until student responds correctly to both the known and unknown items.
   c) Add another known item to the card stack. Repeat the process
   d) Continue until you have reached 9 known items and 1 unknown item.
   e) The unknown item is now considered a known item.
3. Run additional trial
   a) Begin with a new unknown item and the previous unknown item.
4. Session is ended when time runs out or student errors on unknown item three times.

When to use?

Use this intervention if:

- **Student is accurate, but slow with fact recall**
- **Student knows some facts, but not all facts.**

Example Activity



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Resources

Overview
http://www.interventioncentral.org/academic-interventions/math-facts/math-computation-promote-mastery-math-facts-through-incremental-re

PaTAN

Math Vocabulary

1. Exposure
2. Quick Teach
3. **Explicit Instruction** (Vocabulary Routine)

Explicit Vocabulary Instruction

Step 1: Introduce the word

Step 2: Introduce the word's meaning

Step 3: Illustrate the word with examples

Step 4: Check students' understanding

- Ask deep processing questions
- Have students discern between examples and non-examples
- Have students generate their own examples
- Have students generate a sentence (using a sentence starter)

Explicit Vocabulary Instruction

Quadratic

What is it?

A type of polynomial

Can be expression, equation, or function

What are the key features?

Expression/equation must have x^2 or x's being multiplied to give x^2

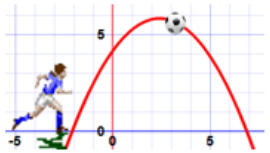
Graph is a parabola

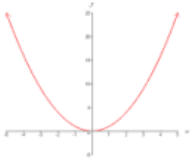
Example(s)

$(x-2)(x+4)$

x^2-2x+3

$x^2-2x+3=8$



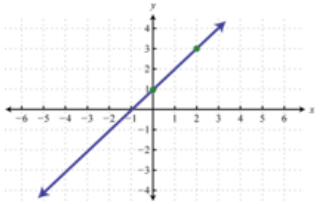


Non-example(s)

$2=3(x-1)$

x^3-2x^2

$(2-3)(x-1)$



Students must practice using the vocabulary!

- Word banks & Word walls
- Sentence starters
- Discussion (peer tutoring)
- Writing
- Reading

Word Problems

1. Attack Strategy
2. Schema-based Instruction

UPS-✓

UPS-✓

Understand

Plan

Solve

Check

Example:
Cognitive
Strategy
Instruction

Schema-Based Problem Solving

Sienna has $70 more than Paul. Paul has $45. How much money does Sienna have?

$$G - L = D$$

$$G - 45 = 70$$

$$G = \$115$$

Greater	
Lesser	Difference

Sienna	
$45	$70

Schema-Based Problem Solving

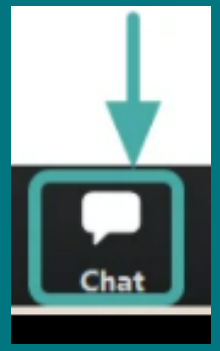
Additive Schema

1. Total
2. Difference
3. Change

Multiplicative Schema

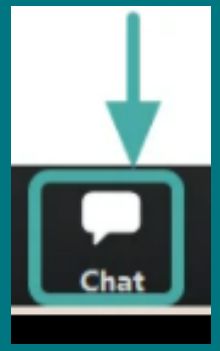
1. Equal Groups
2. A Set Multiplied
3. Ratios/Proportions

Chat



Which intervention are you most likely to use?

Chat



How might you support others to implement these interventions?

PaTTAN
Pennsylvania Training and Technical Assistance Network

Erica Kaurudar
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Jared Campbell
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@justaskjared

Drew Hunter
dhunter@pattan.net

Commonwealth of Pennsylvania
Tom Wolf, Governor

#psychedaboutmath

Break time!

Next session at 10:45

Strategic and Intensive Interventions in Mathematics - Part 2 of 2

Dr. Erica Kaurudar, NCSP
Mr. Jared Campbell
Dr. Drew Hunter
Pennsylvania Training and Technical Assistance Network

#psychedaboutmath

Highlights:

Part 1

Diagnostic Assessment for Intervention Selection

Instructional Hierarchy

Intensification and features of math interventions

Acquisition Interventions

Part 2

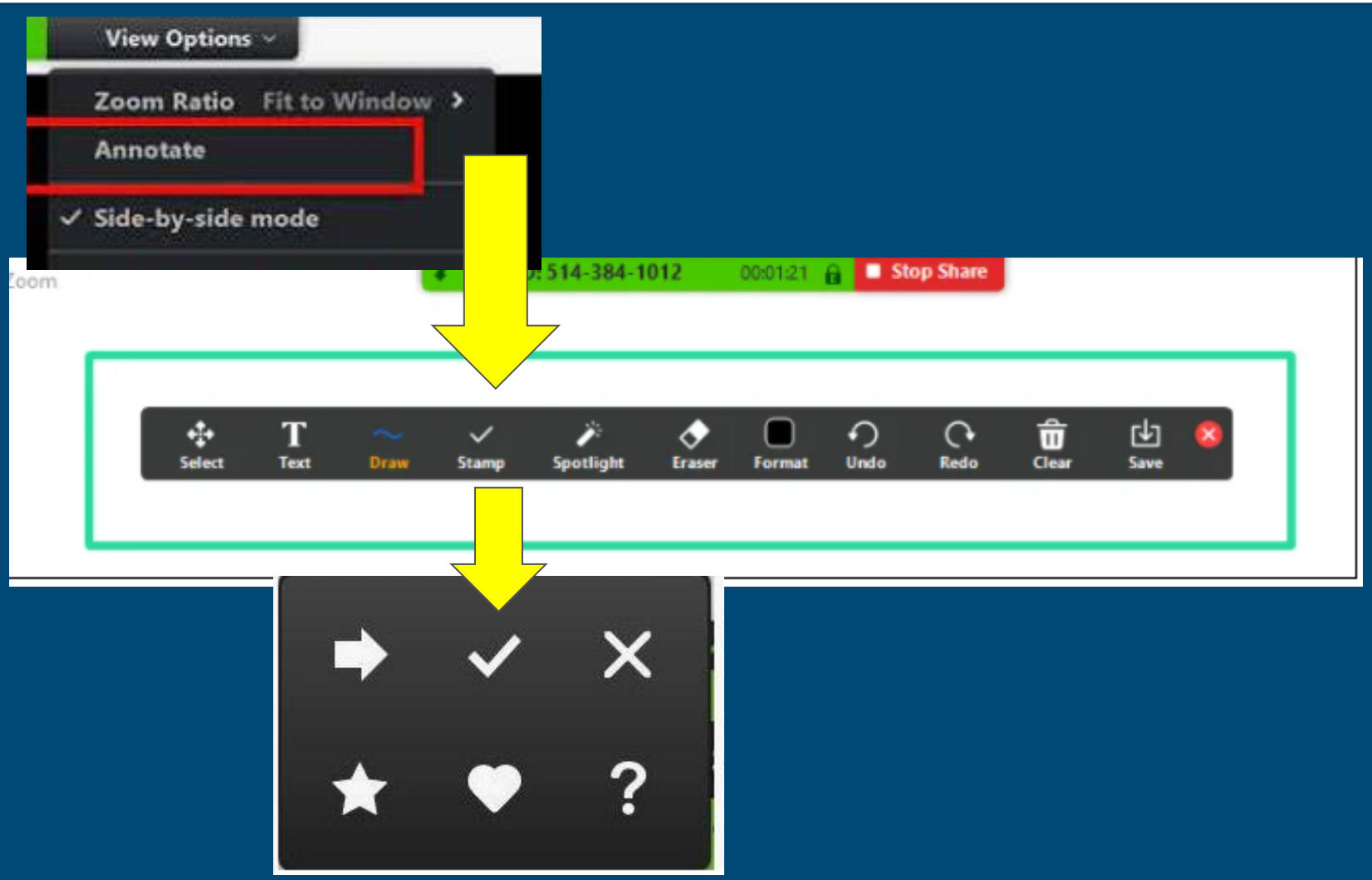
Fluency Interventions

Progress Monitoring

Decision Making

STARLIGHT
Mints

Let’s Stamp!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Stop Share
Select
Text
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Stamp
Spotlight
Eraser
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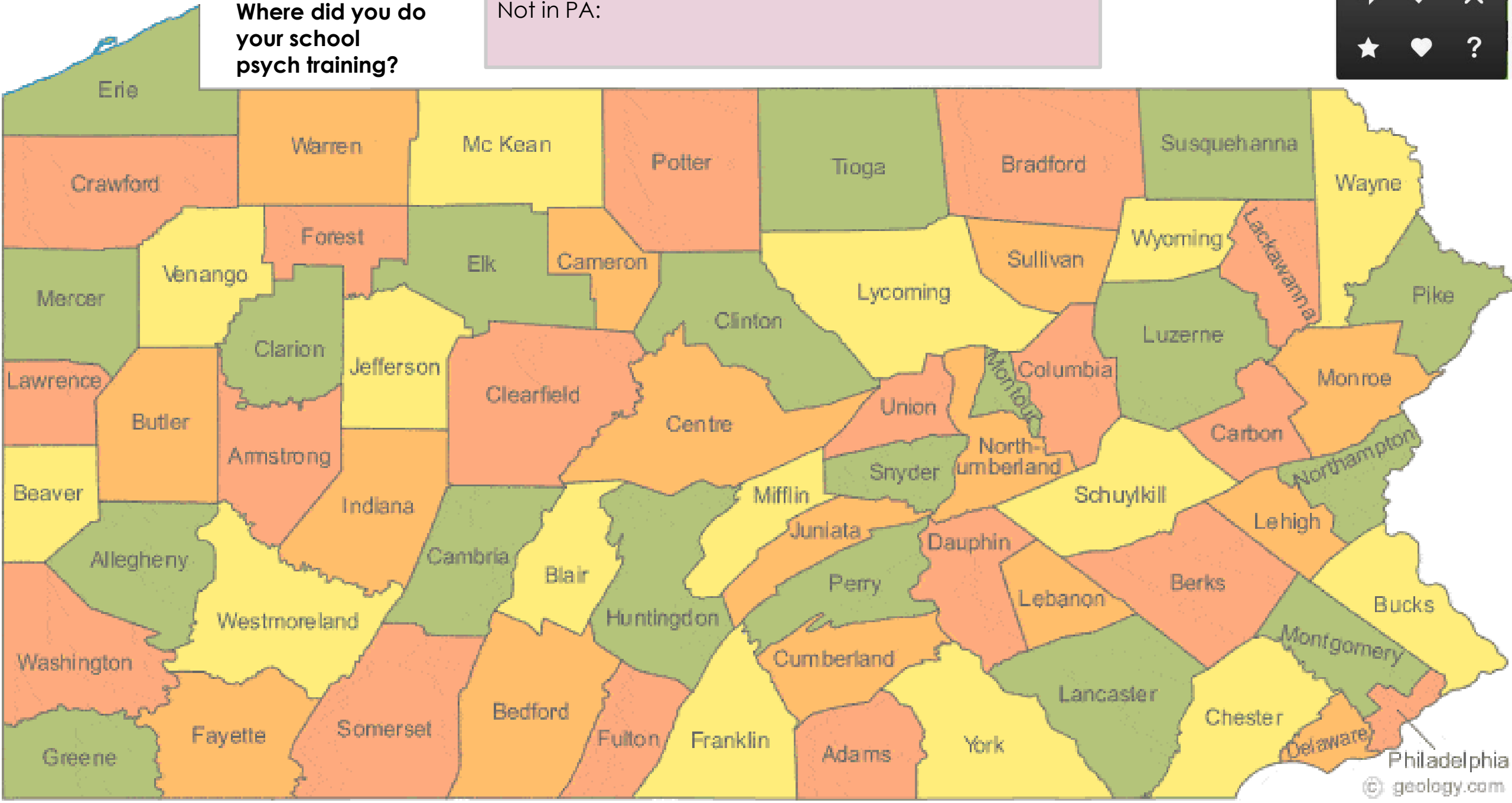
Redo
Clear
Save

Let’s practice!

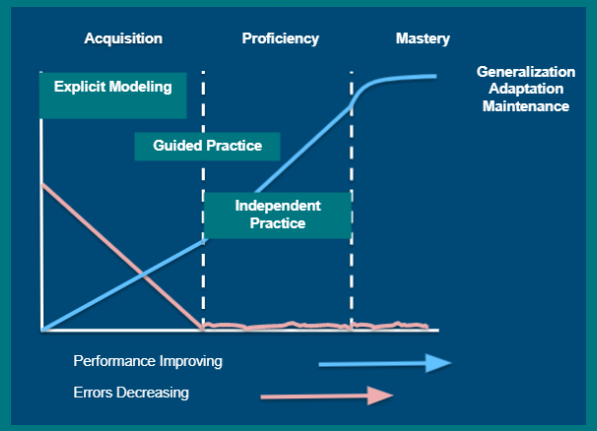
Where did you do your school psych training?

Not in PA:



What best describes your role? You may stamp more than one if it applies to you.

Graduate Student	Higher Ed Faculty	K-12 School Psych Practitioner	Specialized Setting School Psych Practitioner	Administrator	Other



Fluency Interventions

<table>
<tr><th></th><th>Acquisition</th><th>Proficiency</th><th>Generalization</th><th>Adaption</th></tr>
<tr><td>IF

<u>Learning Hierarchy</u></td><td rowspan="2">Haven’t had enough help</td><td rowspan="2">Haven’t had enough practice</td><td colspan="2" rowspan="2">Haven’t had to do it that way before</td></tr>
<tr><td>THEN

<u>Instructional Hierarchy</u></td></tr>
</table>

Adapted from…Haring, N. G., & Eaton, M. D. (1978). Systematic instructional procedures: An instructional hierarchy. In N. G. Haring, T. C. Lovitt, M. D. Eaton, & C. L. Hansen (Eds.) *The fourth R: Research in the classroom* (pp. 23-40). Columbus, OH: Charles E. Merrill.

Mathematics Interventions Aligned to Learning Stages / Instructional Hierarchy

Intervention	Skill(s) Targeted	Individual, Small Group, Classwide Intervention	Time / Dosage	Learning Stage / Instructional Hierarchy
The Great Race	Counting, cardinality (1-10)	Individual, small group	15 minutes (individual) or 25 minutes (small group) 2 x per week for 2 weeks	Fluency
Race to Space	Counting, cardinality (1-100)	Individual, small group	15 minutes (individual) or 25 minutes (small group) 2 x per week for 2 weeks	Fluency
Concrete – Representational-Abstract (CRA)	Conceptual understanding, language / vocabulary, procedural skills for early numeracy, addition, subtraction, multiplication, division, fractions, integers, equations	Individual, small group, classwide	Varied dependent upon skill	Acquisition
Incremental Rehearsal	Basic math facts (addition, subtraction, multiplication, division) Numeral identification	Individual	10-20 minutes per session 2x per week for 4-12 weeks	Acquisition - Fluency
Math to Mastery	Computation (addition, subtraction, multiplication, division)	Individual	10-minute sessions 3 to 5 times weekly for 3 to 8 weeks	Acquisition - Fluency
Drill Sandwich	Basic math facts (addition, subtraction, multiplication, division) Numeral identification	Individual *could be adapted for partner work*	10-20 minutes per session 2x per week for 4-12 weeks	Fluency
Classwide Fluency Building	Numbers and operations, computation skills, word problem solving, procedural fluency	Classwide	Approximately 10-15 minutes daily	Fluency

Cover-Copy-Compare	Computation skills	Individual, small group, classwide	3-10 minutes daily 2 to 5 sessions per week 2 to 16 weeks	Fluency
Detect-Practice-Repair	Computation skills	Individual, small group, classwide	10-12 minutes daily	Fluency *works best for students with higher fluency skills on target skill than those who are dysfluent*
Taped Problems	Math facts; simple and complex computation problems (addition, subtraction, multiplication, division)	Individual *Adapted for Classwide w/ added group contingency	8-15 minutes per session, 3 to 5 times per week for 3 to 10 weeks	Fluency
Explicit Timing	Addition, subtraction & multiplication (single and multi-digit)	Small group, classwide	3-10 minute sessions 2 to 5 times per week 4 to 6 total weeks	Fluency
High-Preference / Interspersed Problems	Complex computation including addition and subtraction with regrouping	Individual, small group, classwide	10 minutes daily 3 to 5 times per week For 3 to 5 weeks	Fluency - Generalization

Classwide intervention is most effective when delivered 4-5 times per week.

What best describes your interest in spending more time on each of the following? Stamp once for each:

Strategy	Full Plate!!!	Just Dessert Please	I want to go home!
The Great Race			
Race to Space			
Cover – Copy - Compare			
Detect – Practice – Repair			
Explicit Timings			

The Great Race

Fluency

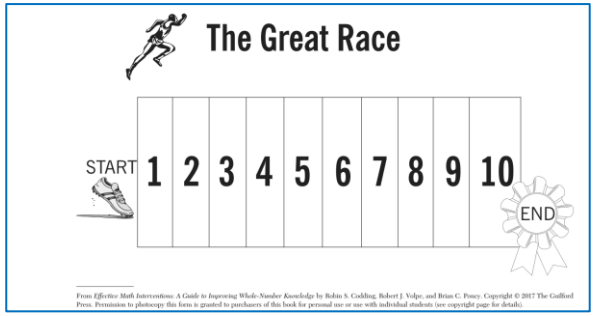
Routine

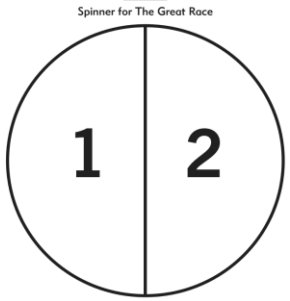
1. Construct a gameboard of 10 horizontally-arranged squares of equal size with numbers 1-10 listed left to right. Ensure “start” is by 1 and “end” is by 10.
2. Create a game spinner by dividing a circle in two halves. One half labeled “1” and the other half labeled “2.”
3. Find items for game pieces representing different colors / shapes (e.g., buttons, beads).
4. Student and teacher each select their game piece.
5. Tell student that you will each take turns spinning the spinner. Whomever reaches the end first wins.
6. Tell the student when it is her turn that he should move his game piece the number of spaces indicated on the spinner (either 1 or 2 spaces).
7. Ask the student to say the number she spun AND to say the numbers on the spaces through which she moved.
   a. For example, if the student was on the numeral 5 and spun 2, she should say, “I am on number 5 and spun a 2, so I will move two spaces.” The student should say, “six, seven” as she moved her game piece.
8. If the student names the numeral incorrectly, provide the student with the correct number and have him repeat the name(s) while moving the token.

When to use?

Use this intervention if:
Student needs practice with counting and cardinality

Example Activity





Resources

Ramani, G.B., & Siegler, R. S. (2008). Promoting broad and stable improvements in low-income children’s numerical knowledge through playing board games. *Child Development*, 79, 375-394.
Ramani, G. B., Hitti, A., & Siegler, R. S. (2012). Taking it to the classroom: Number board games as a small group learning activity. *Journal of Educational Psychology*, 104, 661-672.

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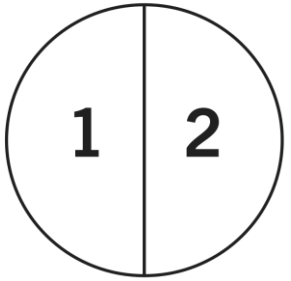
Spinner for The Great Race



Game Board for The Great Race

The Great Race

START

1	2	3	4	5	6	7	8	9	10

END

The Race to Space

Fluency

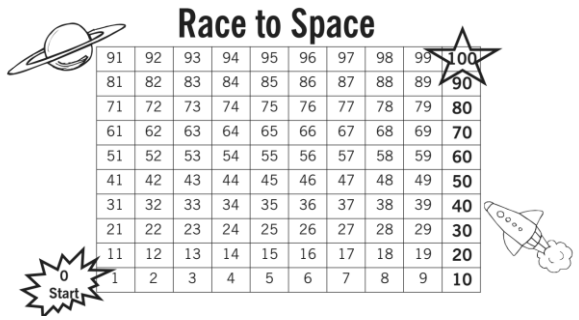
Routine

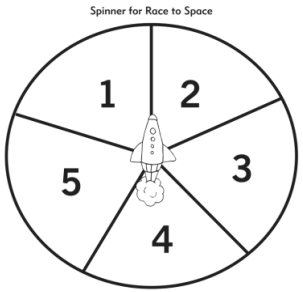
1. Construct a gameboard of 10x10 matrix of squares of equal size with numbers 1-100 arranged from the bottom left (1) to the top right (100). Place the word start with the numeral “0” beside it at the beginning, and place a star or the word “end” by the numeral 100. Background of board gradually darkens as numerals increase, providing a cue for number magnitude.
2. Create a game spinner by dividing a circle into 5 equal sections labeled 1 to 5.
3. Find items for game pieces representing different colors / shapes (e.g., buttons, beads).
4. Student(s) and teacher each select their game piece.
5. Tell students that the purpose of the game is to learn about the numbers 0 to 100. Explain you will each take turns spinning the spinner. Whomever reaches the end first wins.
6. Tell the student when it is her turn that he should move his game piece the number of spaces indicated on the spinner (either 1 or 2 spaces).
7. Ask the student to say the number she spun AND to say the numbers on the spaces through which she moved.
   a. For example, if the student was on the numeral 5 and spun 2, she should say, “I am on number 5 and spun a 2, so I will move two spaces.” The student should say, “six, seven” as she moved her game piece.
8. If the student names the numeral incorrectly, provide the student with the correct number and have him repeat the name(s) while moving the token.

When to use?

Use this intervention if:
Student needs practice with counting and cardinality

Example Activity





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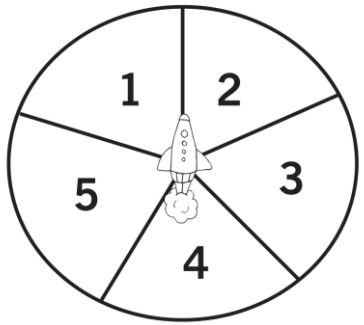
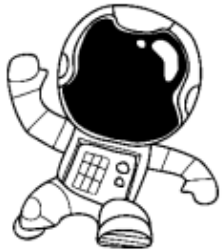
Spinner for Race to Space





Game Board for Race to Space

Race to Space

91	92	93	94	95	96	97	98	99	**100**
81	82	83	84	85	86	87	88	89	**90**
71	72	73	74	75	76	77	78	79	**80**
61	62	63	64	65	66	67	68	69	**70**
51	52	53	54	55	56	57	58	59	**60**
41	42	43	44	45	46	47	48	49	**50**
31	32	33	34	35	36	37	38	39	**40**
21	22	23	24	25	26	27	28	29	**30**
11	12	13	14	15	16	17	18	19	**20**
1	2	3	4	5	6	7	8	9	**10**

0 Start

Cover – Copy - Compare

Fluency

Routine

1. **Develop the Cover-Copy-Compare sheet.**
   a) **Section a worksheet into three columns.**
   b) **Select up to 10 facts for students to practice.**
   c) **Write the facts as correct models on the left column.** (You could use the right column if the student is left handed.)
2. **Student completes the sheet**
   a) **Study the correct fact, practice saying it the fact.**
   b) **Cover the fact.** (by hand or by folding the paper)
   c) **Copy the fact in the next column while saying the fact.**
   d) **Uncover the original, and compare.**
      - **If correct move to next problem**
      - **If incorrect, cross out the incorrect fact and go to step 2a.**
   e) **Continue until all facts are complete.**
3. **Update records/graphs**
   a) **If student correctly complete a facts for 3 consecutive trials**
      - **Update their known facts list to include this item**
      - **Update graph to monitor progress**

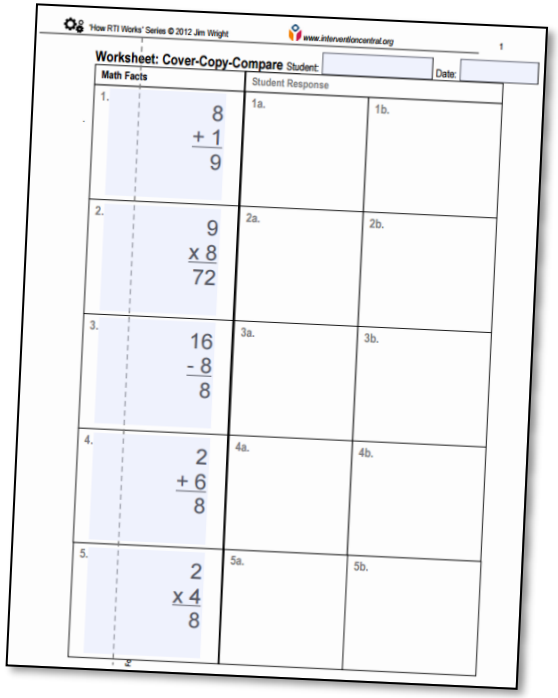
Variation: Instead of writing the facts, students can practice verbal responses.

When to use?

Use this intervention if:

- **Student is struggling to acquire new math facts.**
- **Student is not maintaining accuracy with previously know math facts.**
- **Student is able to replicate the process independently.**

Example Activity



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Worksheet: Cover-Copy-Compare Student: Date:

Math Facts	Student Response	
1. 8 + 1 = 9	1a.	1b.
2. 9 x 8 = 72	2a.	2b.
3. 16 - 8 = 8	3a.	3b.
4. 2 + 6 = 8	4a.	4b.
5. 2 x 4 = 8	5a.	5b.

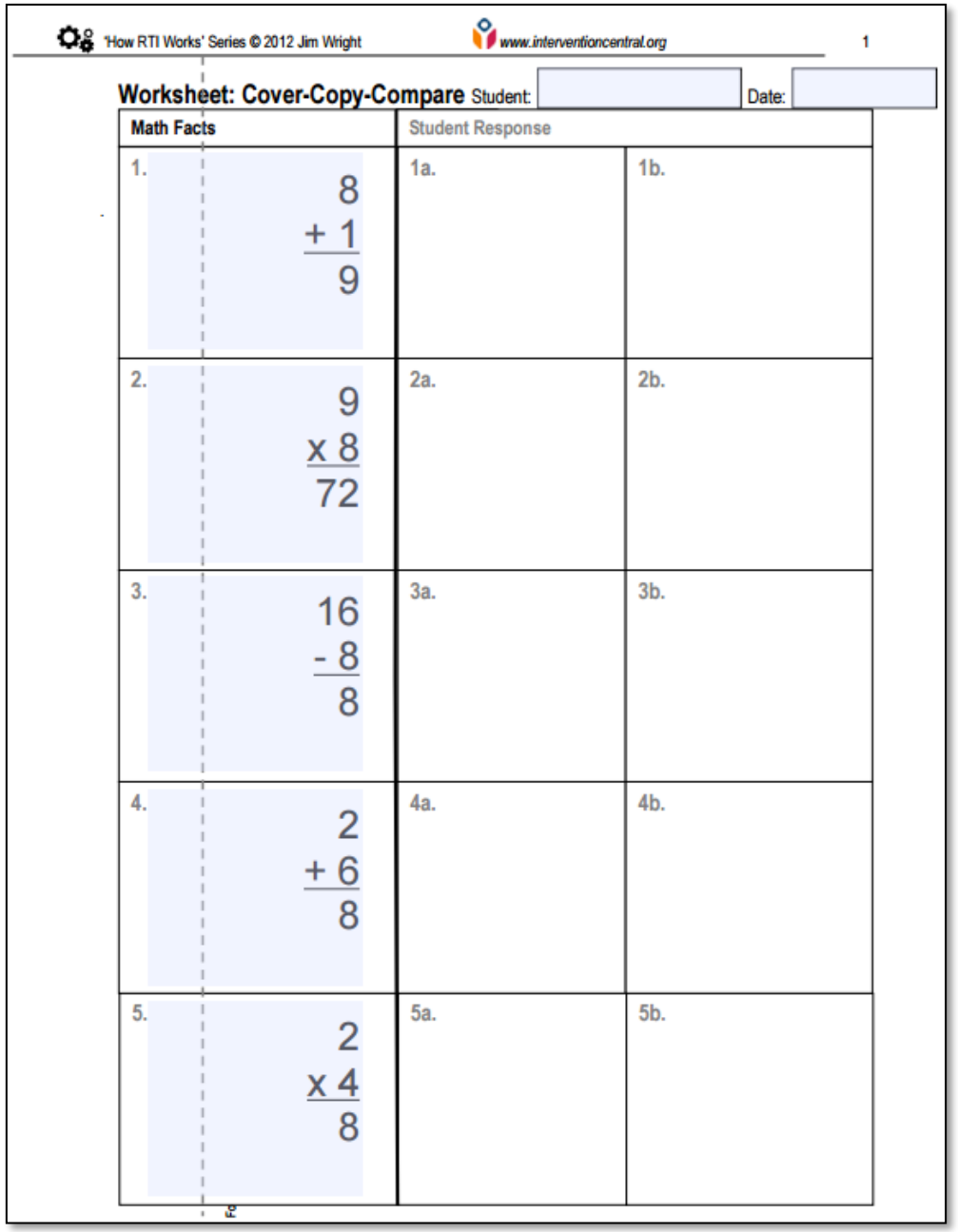
Resources

Overview
http://www.interventioncentral.org/academic-interventions/math-facts/how-master-math-facts-cover-copy-compare
http://www.intensiveintervention.org/chart/instructional-intervention-tools

Known Facts Lists
http://www.interventioncentral.org/sites/default/files/pdfs/pdfs_interventions/CCC_log_form_interactive.pdf

PaTAN

Cover-Copy-Compare Materials



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Worksheet: Cover-Copy-Compare Student: Date:

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3. 16 - 8 = 8	3a.	3b.
4. 2 + 6 = 8	4a.	4b.
5. 2 x 4 = 8	5a.	5b.

Detect – Practice – Repair

Fluency

Routine

1. Detect unknown items
   a) Give students a structured sheet to record their answers.
   b) *Present math fact for 3 seconds, while student writes the answer on their sheet.
   c) Repeat for set of targeted math facts.
   d) At the end of the assessment, display the answer key and have students identify incorrect responses.
2. Practice
   a) Student develops their own CCC sheet
      - Use correct version of their first 5 errors.
   b) Perform CCC procedure.
3. Repair
   a) Teacher provides feedback.
   b) CCC can be repeated.
   c) Other fluency interventions can be applied for additional practice.

Variation(s): Can use a timer and flashcards or PPT and 3 second transitions. Prompts could be provided verbally instead of visually.

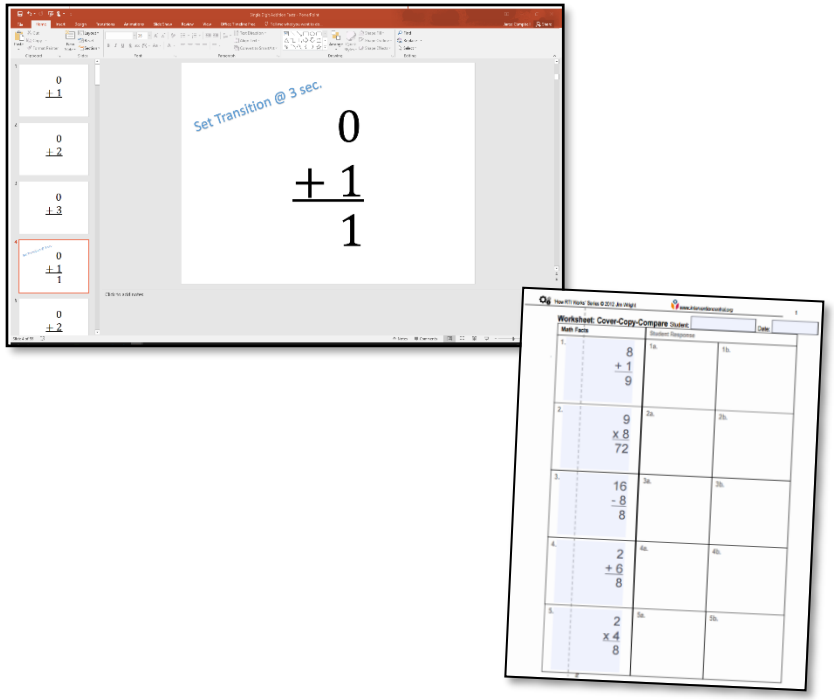
** If using PPT for facts, set the transition duration to 3 seconds.*

When to use?

Use this intervention if:

- Student knows some facts, but not all facts.
- Working with a group of students.
- Cover-Copy-Compare is already in use.
- Need to individualize CCC routine.

Example Activity



Resources

Axtell, Philip K., "Developing Math Automaticity Using a Classwide Fluency Building Procedure for Students with Varying Processing Speeds. " PhD diss., University of Tennessee, 2006.

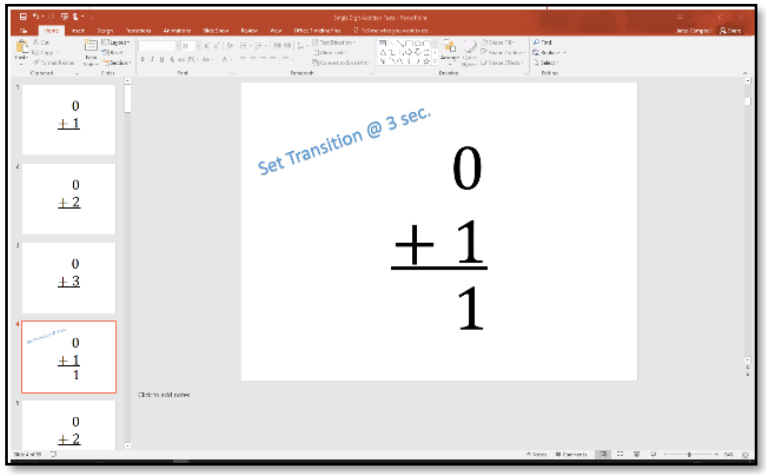
Poncy, B. C., Skinner, C. H., & O'Mara, T. (2006). Detect, practice, and repair: The effects of a class-wide intervention on elementary students' math fact fluency. Journal of Evidence Based Practices for Schools, 7, 47-68.

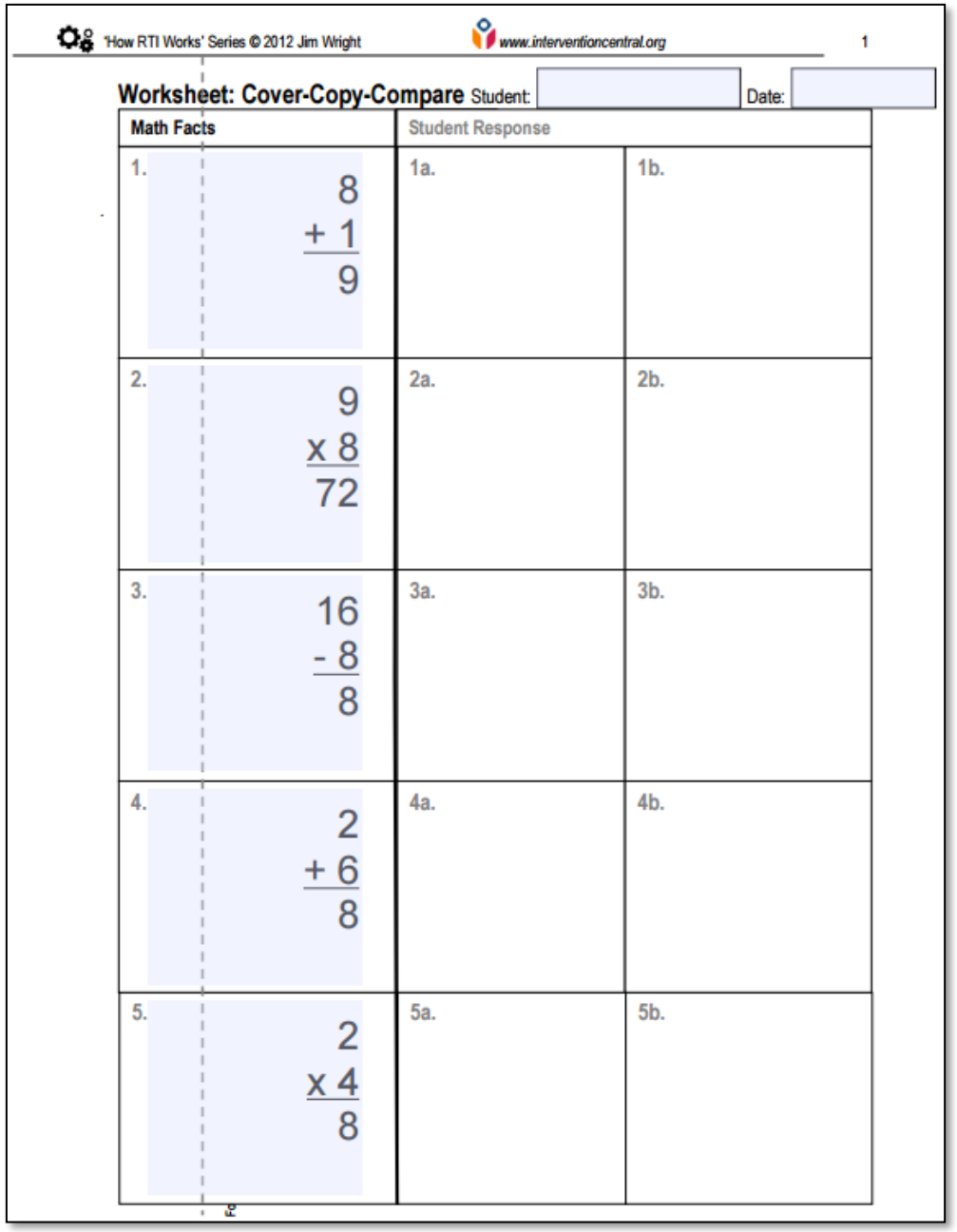
Parkhurst, J., Skinner, C. H., Yaw, J., Poncy, B., Adcock, W., & Luna, E. (2010). Efficient classwide remediation: using technology to identify idiosyncratic math facts for additional automaticity drills. *International Journal of Behavioral Consultation & Therapy, 6*(2), 111–123.

PaTAN

Cover-Copy-Compare Materials

Detect-Practice-Repair Variation with PowerPoint:





'How RTI Works' Series © 2012 Jim Wright www.interventioncentral.org 1

Worksheet: Cover-Copy-Compare Student: Date:

Math Facts	Student Response	
1. 8 + 1 = 9	1a.	1b.
2. 9 x 8 = 72	2a.	2b.
3. 16 - 8 = 8	3a.	3b.
4. 2 + 6 = 8	4a.	4b.
5. 2 x 4 = 8	5a.	5b.

Explicit Timing

Fluency

Routine

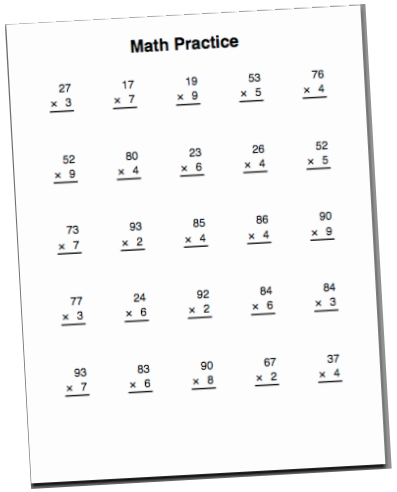
1. Select skill on which student will practice
    a) Develop appropriate skill sequence
    b) Access appropriate practice worksheets
    c) Assess and identify the starting skill
2. Document date, start time, & end time
3. Pass out worksheets or Explicit Timing folder.
4. If doing self graphing, read the following, "class, please open your folder and locate your graph and score from yesterday's practice. Mark your score on your graph and see whether you beat your previous score (pause). Now take out today's math worksheet".
5. Read directions, "Today we are going to complete math worksheets using explicit timing. With explicit timing I am going to give you x minutes to complete as many problems as you can. Your first goal is to complete each problem correctly and to not skip around. In addition push yourself to work as quickly as possible. Ready, begin".
6. Start stopwatch and stop students after x minutes have elapsed.
7. Score probes to inform decision making
    1. Move back a skill (student is inaccurate)
    2. Continue with current skill (student is accurate)
    3. Move to next skill (student has eclipsed mastery criteria)

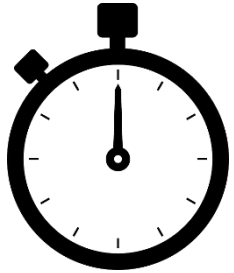
When to use?

Use this intervention if:

1. Student has accurate responding
2. Student has not yet developed fluency

Example Activity





Resources

Codding, R. S., Volpe, R. J., Poncy, B. C., (2017). *Effective math interventions: a guide to improving whole-number knowledge* . New York : The Guilford Press.

http://www.otiss.net/interventions/explicit-timing/

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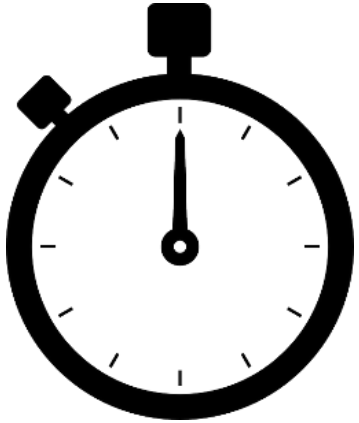
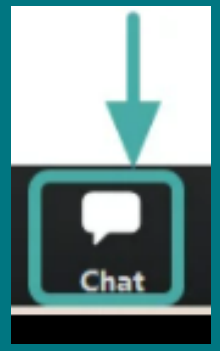


Math Practice

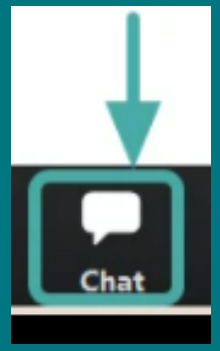
27 × 3	17 × 7	19 × 9	53 × 5	76 × 4
52 × 9	80 × 4	23 × 6	26 × 4	52 × 5
73 × 7	93 × 2	85 × 4	86 × 4	90 × 9
77 × 3	24 × 6	92 × 2	84 × 6	84 × 3
93 × 7	83 × 6	90 × 8	67 × 2	37 × 4

Chat



Which intervention are you most likely to use?

Chat



How might you support others to implement these interventions?

Measuring Response to Intervention

We will look very briefly at evaluating student response to intervention through the lens of Rate of Improvement (ROI), Student Growth Percentiles (SGPs), and Mastery Measurement.

What progress monitoring tool does your district use for math?

DIBELS or Acadience Math	STAR Math	AIMSWeb Plus	Spring Math	NA- Graduate Student	Other (or none, or it's complicated)

Rate of Improvement (ROI)

Progress monitoring is a rigorous assessment technique based on research on applications of repeated measurement techniques featuring brief and frequent measurements.

Important part of the progress monitoring process is the evaluation of student performance over time

Examples: AIMSWeb, DIBELS, Star

ROI

Important part of the progress monitoring process is the evaluation of student performance over time

Behavior over time is reflected in **rate of change or slope**

Slope calculation needs to be for **typical/benchmark** performance as well as the **expected** and **actual** levels of the student's performance

ROI Terms

Important Terms:

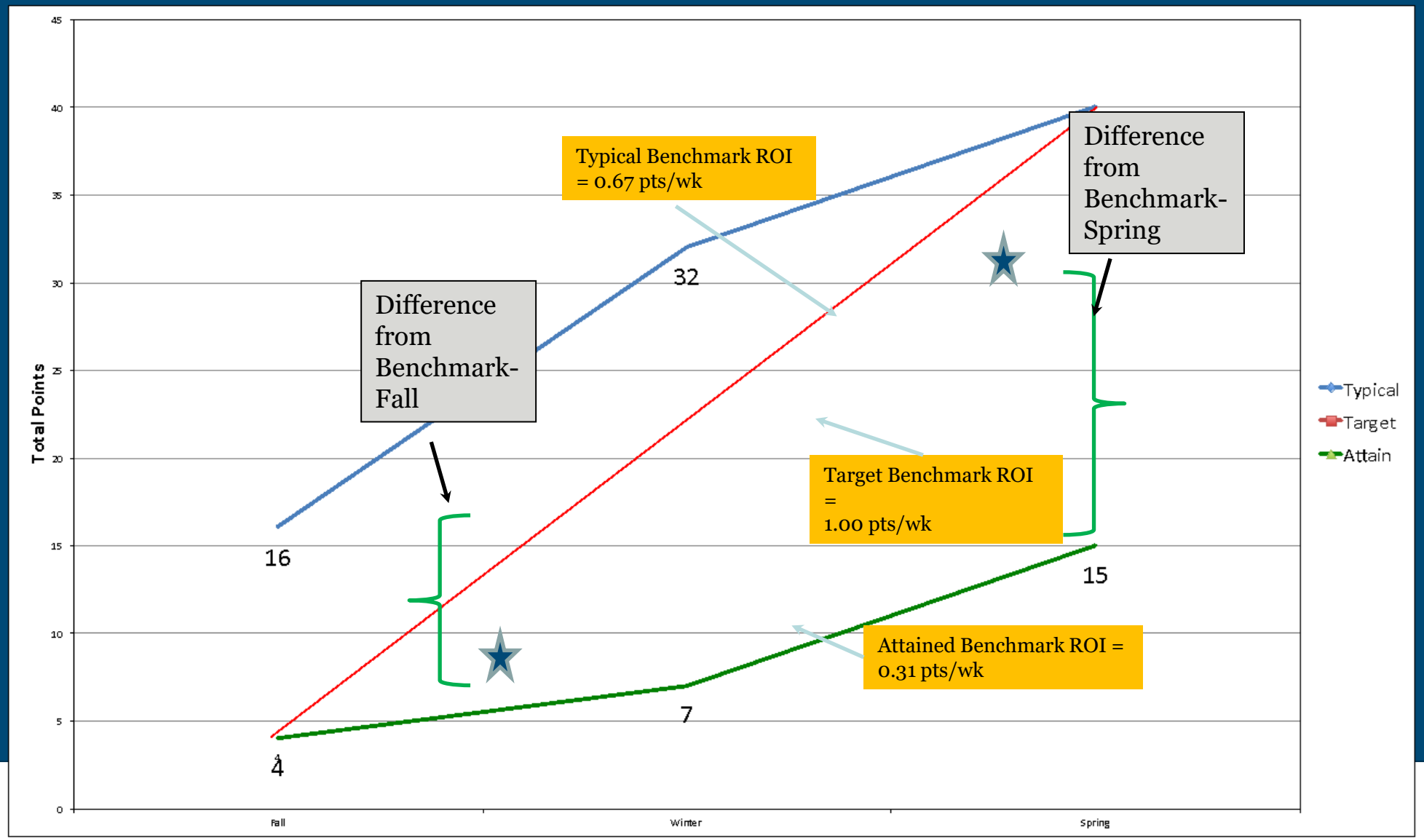
Typical ROI=From benchmark to benchmark

Target ROI=From starting score of student to benchmark of typical benchmark

Attained ROI=From starting score of student to ending score of student

Shapiro (2011)

Grade 2 Student – How Low? How Slow? Example



Results- Benchmark ROI

- Typical ROI
  - **Fall to Spring** **(40 –16)/36 = 0.67 pts/wk**
- Target
  - **Fall to Spring** **(40 –4)/36 = 1.00 pts/wk**
- Attained ROI
  - **Fall to Spring** **(15 – 4)/36 = 0.31 pts/wk**

- Student making progress over time (increasing)
- Student moving at slower rate than typical peers
- Student NOT closing the gap between

Shapiro

Best Practice: Ordinary Least Squares (OLS) aka Progress Monitoring ROI

Uses **linear regression**

-Mathematical process for establishing the straight line that cuts through ALL the data points & establishes LINEAR TREND in the data

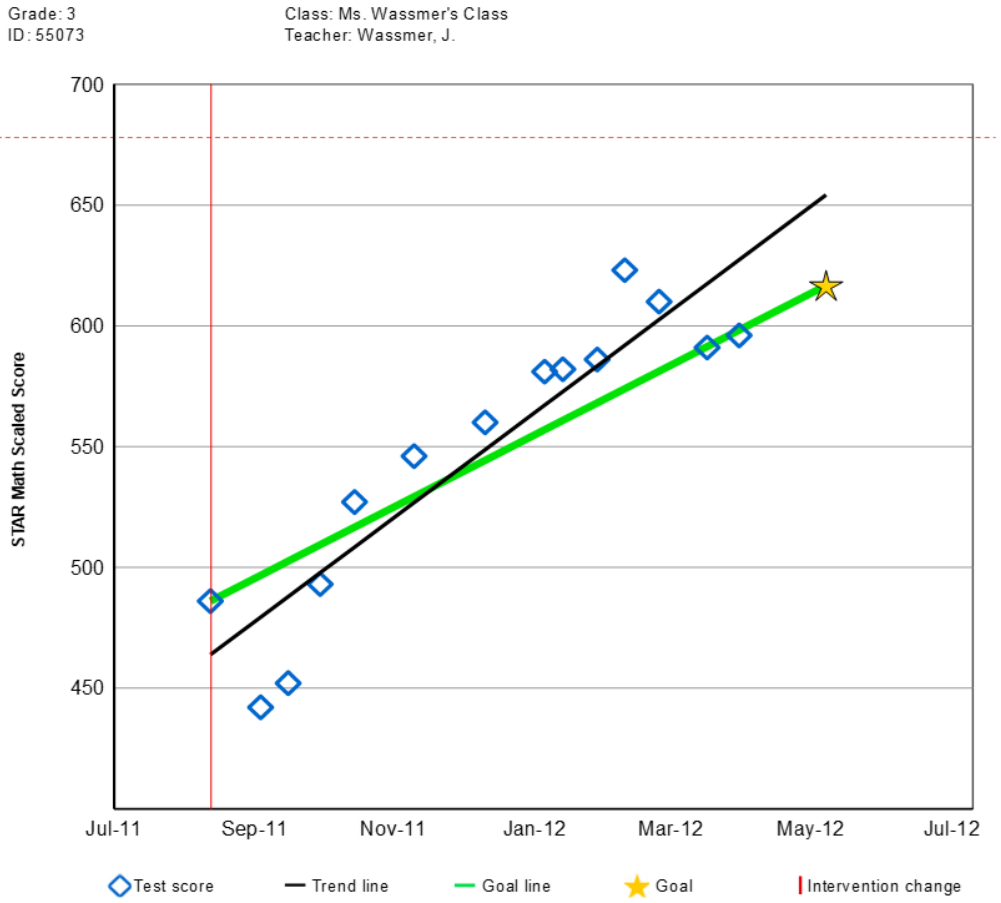
-**AIM** line (expected performance) **TREND** line (actual performance)

- Takes into account ALL data points in the series
- Requires mathematical calculation (let software do it)!
- Some software (e.g., DIBELS, AIMSWeb) has capability to calculate it for you
- EXCEL can do it too!

Shapiro

Example OLS / Progress Monitoring ROI

Sample PM from STAR Math



Angelina's Current Goal

Goal: 616 SS 54 PR (Moderate) **Goal End Date:** 6/2/2012 **Expected Growth Rate:** 3.4 SS/Week

Student Growth Percentiles (SGPs)

Student Growth Percentiles (SGP)s are calculated from triannual (3 x year) assessments and provide a normative comparison of growth to a student's academic peers (e.g., students who started at the same/similar level).

Reported as percentile rank. Example: SGP of 50 is interpreted to indicate the student's growth was equal or better than 50% of academically-like peers. (Examples: Acadience, AIMSWeb)

aimsweb Plus Examples: SGPs and Descriptors

Very low: 1st to 10th percentiles

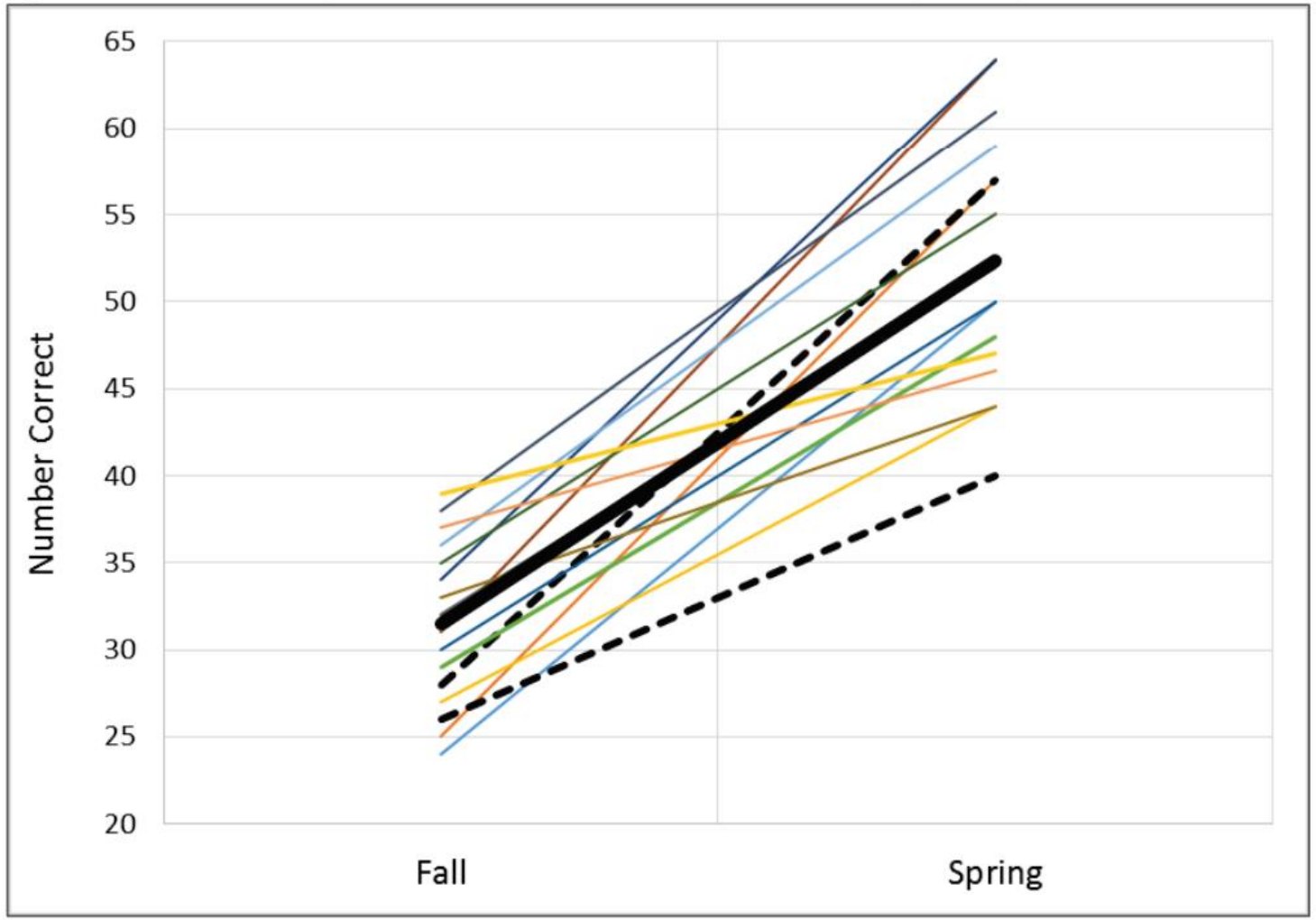
Low: 11th to 25th percentiles

Average: 26th to 74th percentiles

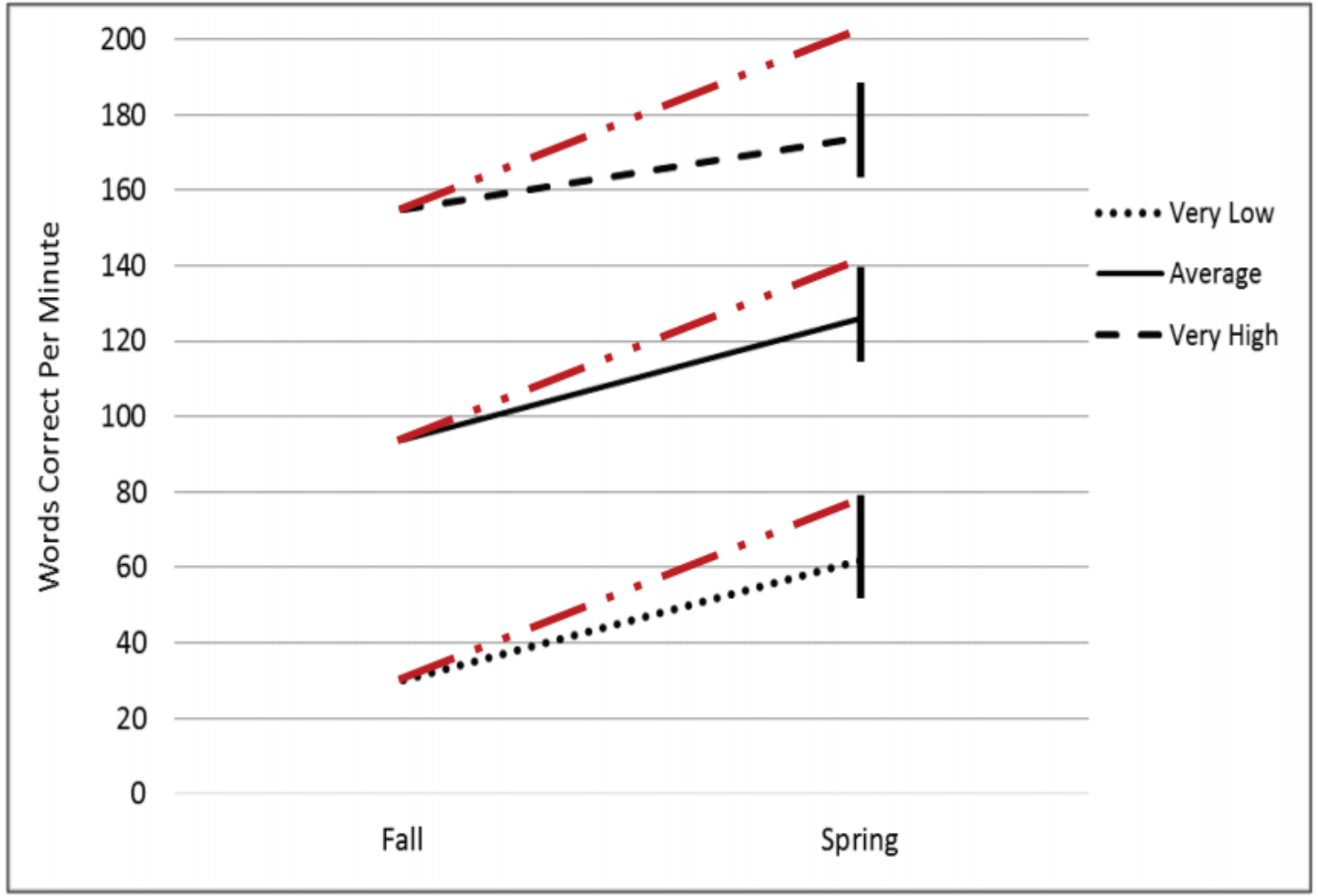
High: 75th to 89th percentiles

Very high: 90st to 99th percentiles

aimsweb Plus Example



aimsweb Plus Example



Goal Setting Example

Insufficient (SGP ≥ 50): The ROI is below average and will not improve the student's percentile rank enough to sufficiently close the performance gap.

Closes the gap (SGP of 51–84): The ROI is above average and the goal score will improve the student's percentile rank.

Ambitious (SGP of 85 –97): The ROI is well-above average and the goal score will substantially improve the student's percentile rank.

Extremely Ambitious (SGP ≤ 98): The ROI is greater than 97% of the ROIs in the national sample for students with a similar initial score and is rarely achieved in typical settings.

aimswebPlus SGP Guide (2017)

Mastery Measurement:

Measurement of student performance of a specific target skill

Curriculum-Based Assessment Mathematics
Single-Skill Computation Probe: Student Copy

Student: Date: _____

29 +64	28 +69	46 +46
54 +27	74 +18	51 +29
77 +26	16 +67	27 +65

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Spring® MATH

Assessing
Response to Intervention/Student Growth

Individual with Goal (Benchmark)
Peers with Goal (Benchmark)
Individual with Peers

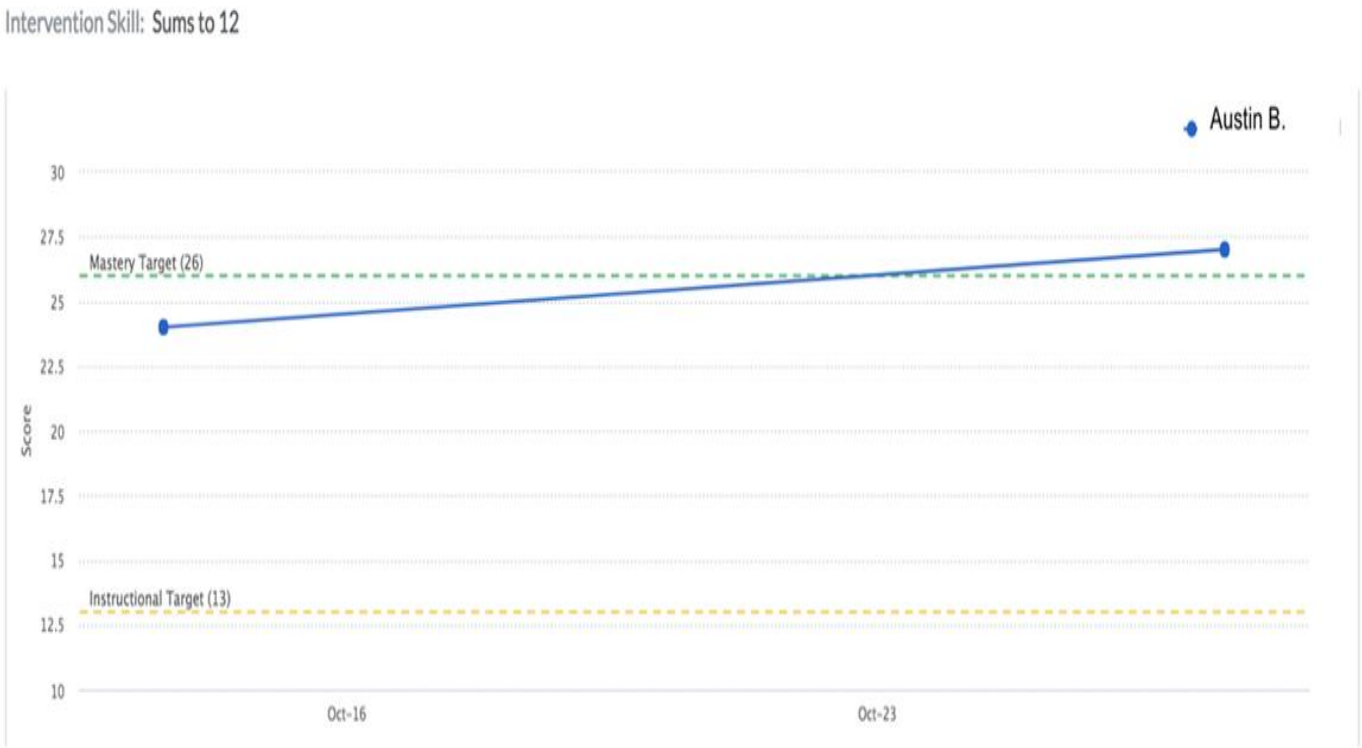
Individual Growth: Goals for Austin

- Austin will increase levels in intervention at least every 4 weeks with intervention consistency score of at least 80%.

- Austin's post-intervention performance will be in the not-at-risk range on the first goal skill for which he received intervention.

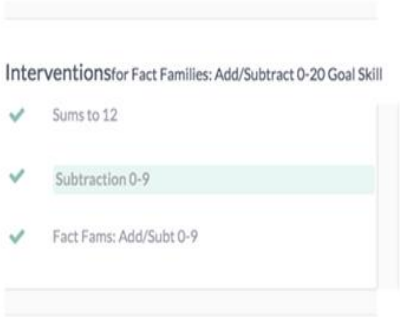
- Austin will perform outside of the risk range on subsequent screenings.

First Individual Intervention Skill, Adequate Growth

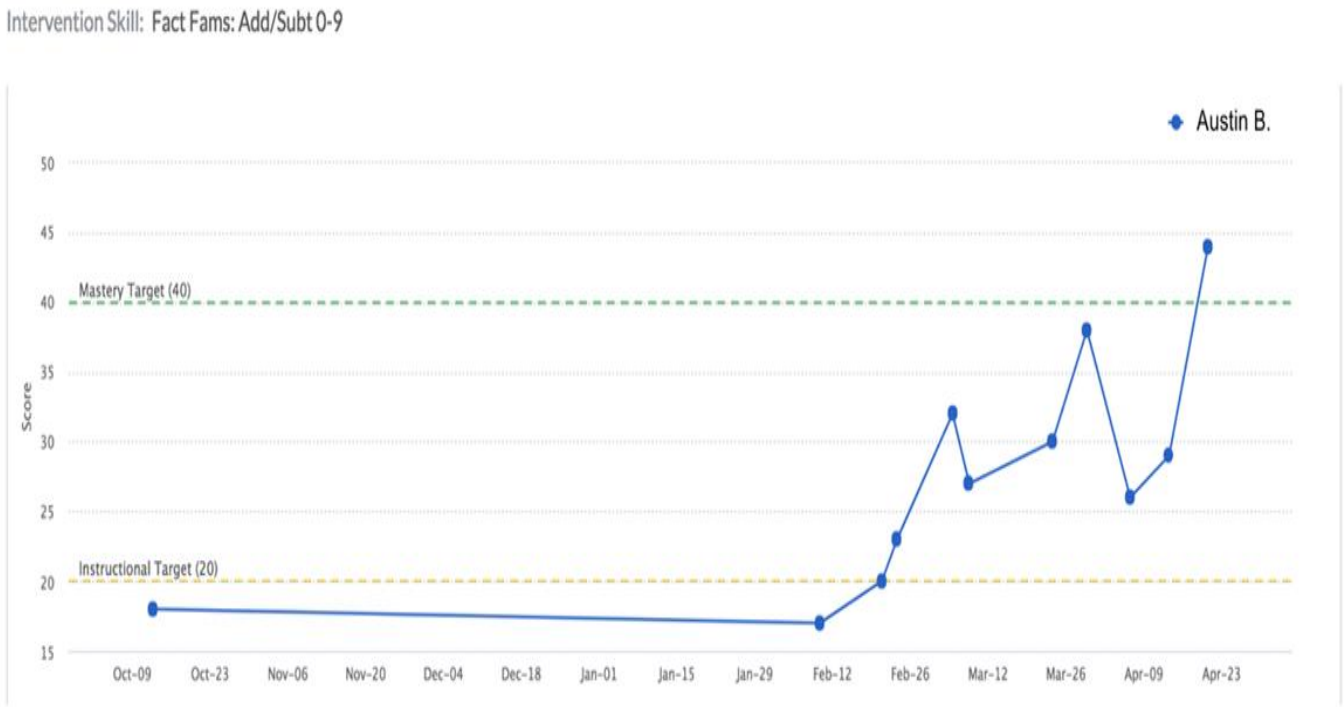


Spring Math

Second Individual Intervention Skill, Adequate Growth

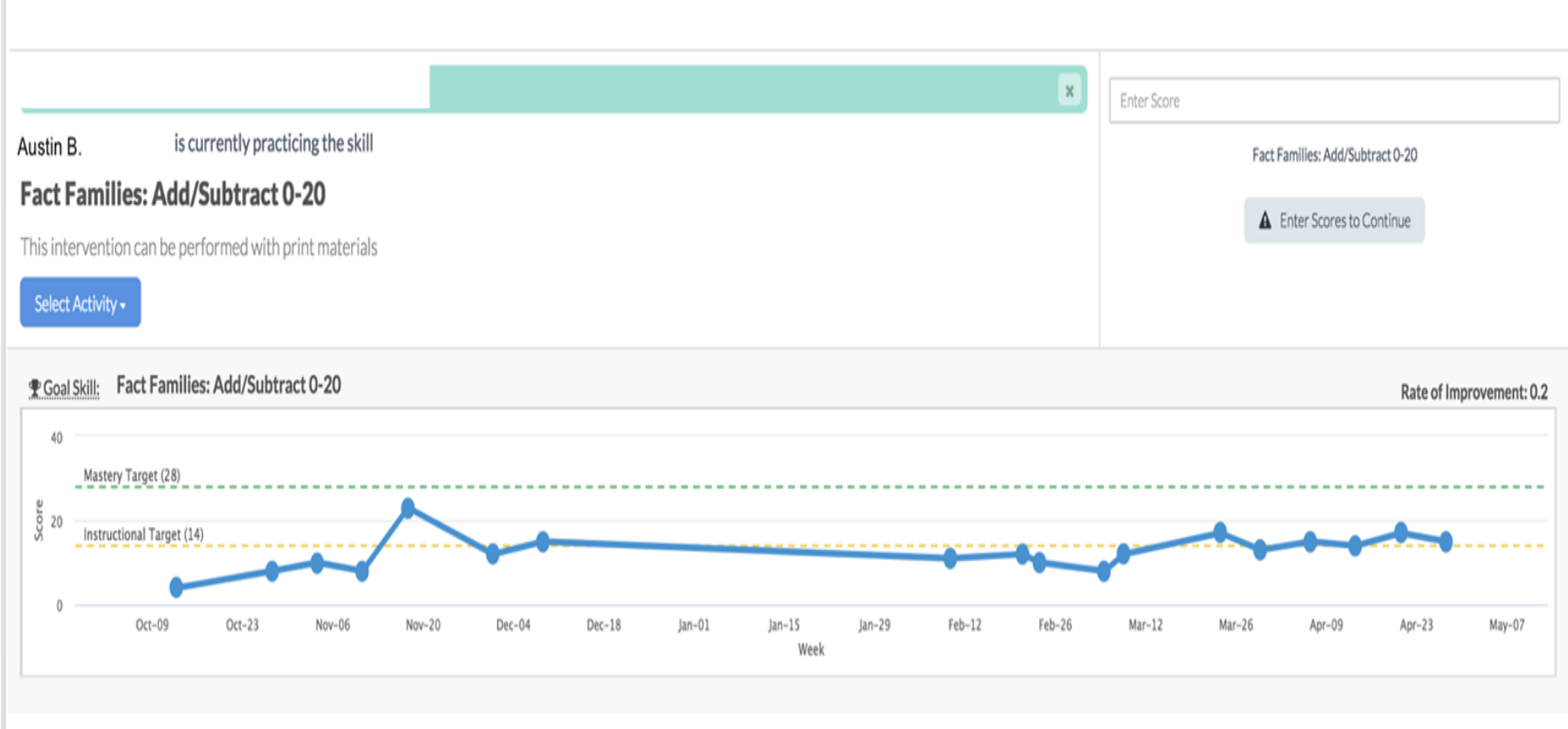


Third Intervention Skill, Slow Growth: Met Mastery Goal with 9 weeks of Individual Intervention



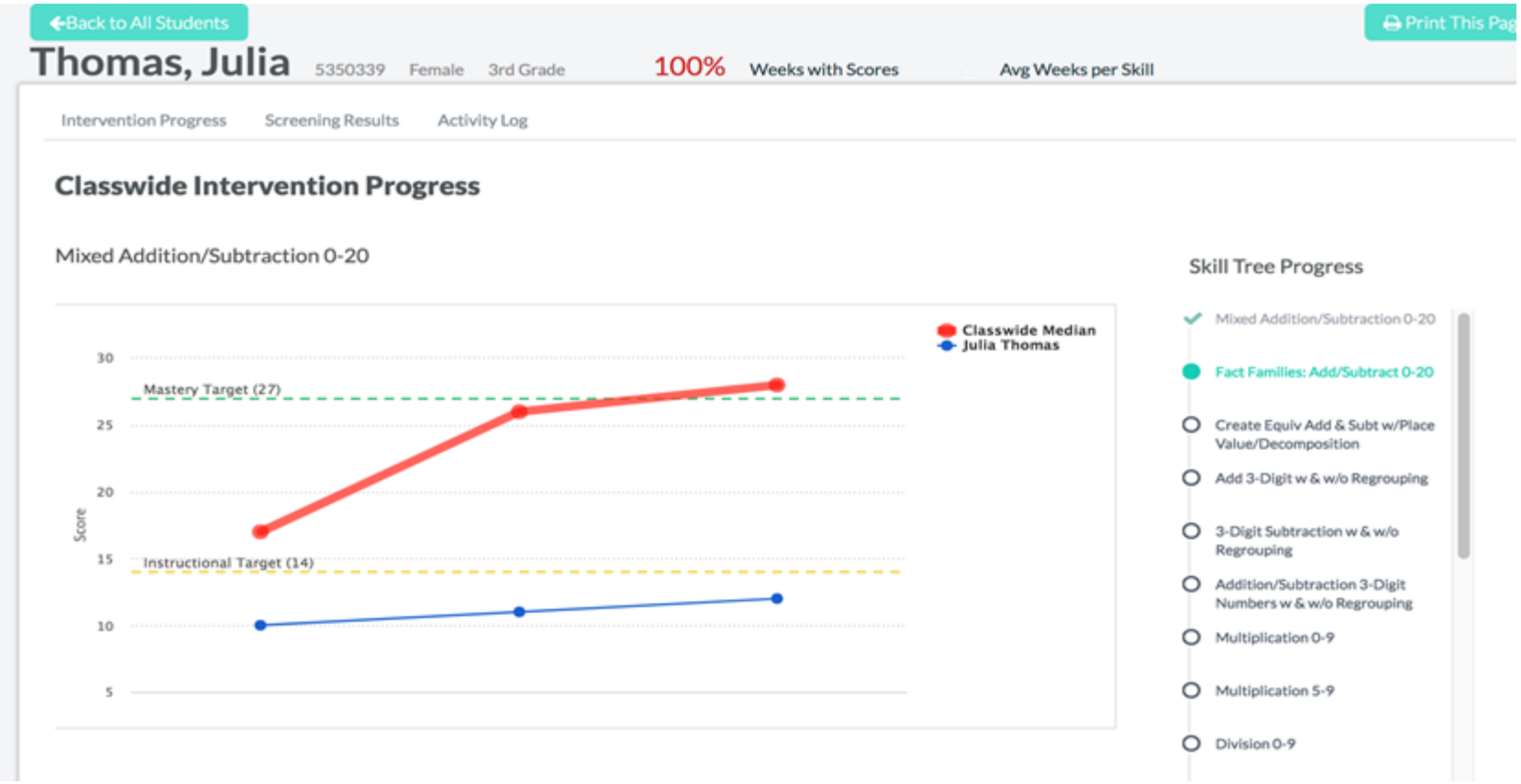
Spring Math

Poor Generalization to Goal Skill. Inadequate Growth on the Goal Skill.



Spring Math

Student Progress Less Steep as compared to Classwide Median



What factors might need to be considered before making intervention adjustments?

Considerations Before Making Intervention Adjustments

-Was student present for **85% to 95%** of intervention sessions?

-Was the intervention being delivered with the amount of time and intensity as planned?

-Were **treatment integrity/fidelity data** collected?

- -Direct observation of intervention
- -Review of permanent products
- -Interventionist self-report of completed steps

DiGenarro Reed & Codding, (2014)

Considerations Before Making Intervention Adjustments

-Was the student being provided with intervention in the **correct level** of **instructional materials**?

-Correct subskill?

-Learning stage considered? (e.g., acquisition, fluency, maintenance / generalization)

Riley-Tillman, Burns, & Gibbons (2013)

Evaluating Student Progress and Making Intervention Adjustments

Little / No Improvement	Slow Improvement	Meets Goal Line	Exceeds Goal Line
Teach Prerequisite Skill	Increase Dose	Make No Changes	Adjust Target Skill
Change Assistance	Change Assistance	Reduce Assistance	Fade Treatment Dose
Change Grouping	Modify Intervention		Remove and Monitor
Change Intervention			

Adapted from Codding, Volpe, & Poncy (2017).

Q & A

Let's chat!

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